

# SCHOOL SCIENCE AND MATHEMATICS

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VOL. XXXIV, No. 4

APRIL, 1934

WHOLE NO. 294

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## SOME EFFECTS OF THE DEPRESSION UPON THE TEACHING OF SCIENCE\*

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It is platitudinous to state that education has suffered disastrously as a result of the economic depression. In despairing impotence we have had to witness the loss one by one of advances which have been gained through heroic struggle over a period of many years. At the mercy of the embattled taxpayers, themselves harassed and panicky, we have had to endure the elimination and the consolidation of teaching positions with consequent mounting unemployment of teachers; we have seen the program of studies robbed of the so-called "frills" which may later come to be recognized as among the most generally valuable of all subjects taught; we have seen reduction of salaries more drastic than we had deemed possible even in our moments of most pessimistic foreboding—in fact, reductions to a point in many cases below the level of decent living; we have found it necessary to take on heroic teaching loads. These and other major catastrophes have brought home to us the unwelcome realization that for teachers the "code" is working in reverse, their portion being longer hours, greatly increased loads, and appallingly reduced income. We have been forced to conclude that teaching is the "forgotten calling" and the teacher the "forgotten man."

\* Delivered at the general session of the Central Association of Science and Mathematics Teachers at Chicago, December 1, 1933.

But it is idle merely to recount at length the disasters which have overtaken education. Moreover it is inexcusable to follow the line of least resistance and therefore to remain passively awaiting the return of better days. Better days will surely come ultimately; but they are almost certain not to bring with them a restoration of pre-depression conditions. Certain aspects of teaching are permanently altered, and however much we may deplore these changes and long for the return of the "good old days" we must face present realities. Prudence dictates that we soberly take stock of the situation as it now exists in order to salvage from the wreckage what we may.

We teachers of science are of course chiefly concerned with those phases of the present situation which most vitally affect our own particular work. But it is impossible to separate our problems from those of teachers in other fields. Most of the conditions which vitally affect us apply with equal force to the situations of our colleagues in mathematics and in every other subject. The effects of the depression upon the teaching of science must for the most part mean the effects of the depression upon teaching in general. With this realization clearly in mind let us consider some of the conditions confronting us which seem of most immediately pressing concern and for which we must if possible devise remedies.

*During the years of the depression there has been a progressive increase in class size.* Until the past year the North Central Association recommended an average pupil-teacher ratio not to exceed 25:1. Yet in 1930, 11 per cent of the Association schools had a pupil-teacher ratio in excess of this recommended average; in 1931, the percentage had risen to 14, and in 1932, to 21 per cent. Last year the Association permitted schools to have a 30:1 ratio provided they had met all other standards but had found it impossible to keep the pupil-teacher ratio to 25:1. At the close of school last June it was found that nearly 32 per cent of the Association schools or about three times as large a percentage as in 1930 had a pupil-teacher ratio exceeding 25:1, and that nearly 7 per cent had a pupil-teacher ratio exceeding 30:1. When it is remembered that this pupil-teacher ratio is based upon average enrollment, it is obvious that there must be many classes of heroic size. Science classes enrolling more than forty pupils are now common, and classes with more than sixty pupils are encountered here and there.

Some of our educators have welcomed the movement to in-

crease class size. A few have even pronounced it to be a "blessing in disguise." They argue that large classes not only provide challenging opportunities for skillful teaching but also make it imperative to evolve more effective teaching techniques and skills. A goodly number of research investigations have been published dealing with the relative effectiveness of various instructional procedures in small and large classes. The findings of many of these studies have been interpreted as demonstrating superior advantages of larger over small classes. An imposing array of statistical data in the form of test results have been marshalled in support of these conclusions.

But let us not be stampeded into a too ready and insufficiently considered acceptance of these results. Let us remember that educational research in spite of its notable achievements is still in its infancy. Let us recall that we are able as yet to test only a few of the outcomes of any course; that most of the tests we must use in measuring outcomes have as yet been developed scarcely beyond the crude experimental stages and that these tests are worthy of considerable question with respect to their validity, reliability and other desirable qualities. Let us also remember that there are in any classroom situation many "intangible factors" which as yet we have no way whatever of evaluating. Surely it is reasonable to assume that until at some future time unassailable investigation can establish the optimum class size for each subject, there must be certain humanizing and socializing values which the pupil may gain from membership in a small class which if provided at all in a very large class must be present to a much smaller extent.

Let us keep in mind, however, that any movement which effects economies is certain to be a popular one with taxpayers and with superintendents and board members responsible for the economical conduct of the schools. Because large classes can be administered more cheaply their apparent advantages are likely to be magnified and their obvious disadvantages minimized. Therefore the trend toward large classes is, for a time at least, certain to gain momentum, however much many of us may deplore it. We teachers of science must therefore be alert to devise ways and means of improving our teaching methods to the end that we may teach with increasing effectiveness in spite of large classes. We need for example to increase our knowledge of techniques for directed study and our abilities to impart a working knowledge of these techniques to children. We need

to study the art of skillful questioning to the end that we can include in the work-sheets and in class discussions stimulating and thought-provoking questions appropriate to all levels of ability. We need to improve our own manipulative skills to the end that we may attain greater perfection of demonstration and accompanying exposition. We need to concentrate upon the problem of devising new and ingenious types of tests to measure outcomes other than the mere memorization of factual material—for example, the ability to apply the methods of scientific investigation to new situations, the ability to apply scientific attitudes to life situations and the ability to apply principles of science in out-of-class emergencies. We need to be more than ever alert to new developments reported in educational literature, especially to those reports of careful research which evaluate teaching techniques and procedures in a scientific way. In short, we must become classroom practitioners of extraordinary efficiency, having the same eager and purposeful desire for supremely skillful craftsmanship which characterizes the best members of the medical, legal, engineering and other professions.

*As a result of the depression there has been a progressive decrease in individual laboratory work.* It will be recalled that at about the beginning of this century the individual laboratory method was the commonly accepted one. Teachers frequently deviated from it by allowing pupils to work in pairs or in small groups; but the practice of teacher demonstration as a substitute for pupil manipulation was comparatively rare. However the high school movement beginning about 1892 and lasting almost to the present time brought about a phenomenal influx of pupils into the high schools. The magnitude of this high school movement becomes readily apparent from the statement that it necessitated on the average the building of one new public high school every day for more than thirty years. The problem of furnishing laboratory courses for the increasing hordes of pupils became increasingly acute. It soon became obvious that individual experimentation of the sort in which each pupil himself performs every experiment had become prohibitively expensive. In this emergency the harassed administrators welcomed the demonstration method as the only solution of this problem.

Between the years 1912 and 1927 a considerable number of research investigations were carried on for the purpose of deter-

mining the relative values of the individual and the demonstration methods. The data secured from practically all of these studies were at the time of their reporting interpreted as indicating if not superior at least equal advantages in favor of demonstration by the teacher over manipulation by individual pupils. It would be neither practicable nor profitable here to attempt an analysis of the techniques and findings of these investigations; such analyses have been ably made by a number of experts. But it is interesting to note that the apparent superiorities of the demonstration method were hailed with enthusiasm by administrators who saw in the elimination of the individual method an enormous saving of expense for laboratory equipment. In 1928 Horton<sup>1</sup> using a more elaborate and convincing technique than had hitherto been applied to the problem of the relative merits of the two methods of conducting laboratory work, secured data which indicated certain marked superiorities of the individual method. With their convictions concerning the potential values of the individual method strengthened by this investigational evidence, experts in the field of the teaching of science recommended a laboratory program providing both individual and demonstration experimentation. Such was the situation at the beginning of the depression. During its progress, as the growing necessity for curtailing expenses has become everywhere more imperative, the individual method has again come to be considered impracticable by an increasing number of those who are responsible for financing the schools. To the consternation of teachers of science both in secondary schools and in colleges, the individual method has been eliminated entirely in an alarming number of schools, its place having been taken by the demonstration method or by classroom activities which include no use of apparatus whatever.

The necessity for the curtailing of expense for laboratory equipment is not however the only reason for the decrease in the use of the individual method. Faced with the pressing necessity for reducing the teaching staffs, many administrators have been forced to eliminate the double laboratory period which has long been deemed essential to the proper conduct of the individual method. In many schools, therefore, such pupil experimentation as is done must be accomplished in a single forty-five minute period. In other schools, the science teachers, de-

<sup>1</sup> Ralph E. Horton, *Measurable Outcomes of Individual Laboratory Work in High School Chemistry*. New York: Teachers College, Columbia University, 1928.

spairing of accomplishing effective laboratory work in such restricted time have made no effort to retain individual pupil experimentation. In some schools the science work has retrograded to its status of fifty years ago with the chief emphasis upon information and appreciation.

Now thoughtful observers of the teaching situation throughout the country have long known that sooner or later the double laboratory period would be eliminated. It has long been doomed to succumb eventually to persistent attacks from two quarters, (1) from the classroom teachers of other high-school subjects who insist that science is no more entitled to a double period than their own subjects and that if they were given a double laboratory period for work in English, mathematics, history, and so on, they would no doubt be able to show quite as much value for the time invested as have the science teachers; and (2) from heads of schools who oppose the double laboratory period on the grounds that it is difficult to administer both because it is difficult to fit into the program schedule and because it usually demands the use of two rooms. Recognizing the strength of this opposition teachers have for some time been carrying on investigations for the purpose of learning how to retain the individual method even though restricted to a single laboratory period. But the depression has precipitated the issue long before we are ready to meet it. What is to be done under these circumstances? Are we teachers of laboratory science to fold our hands in impotence or indifference? Certainly not. Believing as all of us must that there are unique values in the individual method which our courses must somehow be made to provide we must devise ways and means of retaining it in spite of all handicaps.

Two steps are fairly obvious. First, it is clearly unreasonable in many schools to expect to conduct any laboratory course, even chemistry, by the "even-front" method. Except for the simplest exercises it will no longer be practicable to furnish enough duplicate sets of apparatus that all members of the class can perform the same experiment simultaneously. Instead, a few sets for each laboratory exercise must serve. The teachers must plan their work, as many have already done, so that different pupils during the same laboratory period will be working upon a considerable variety of experiments, exchanging apparatus and problems upon the completion of an exercise. This idea is by no means new, but it is due for a much wider use. The

introduction of such a plan involves much extra work on the part of the teacher in providing a sufficiently large assortment of experiments which can be performed at the same time, and in providing additional ones in reserve to be performed by the abler pupils who finish first. It also involves much more diversified teaching and much more individual help. But the task is not impossible of accomplishment.

Second, if we are to retain individual experimentation in the laboratory program restricted to forty minutes or even fifty-five minutes in the clear, obviously many of the traditional experiments will need to be either discarded or revised. Any thoughtful observer of the traditional two-hour laboratory period is likely to be convinced that often much time is wasted by the pupils. One investigation of merit<sup>2</sup> produced data which indicated that pupils in chemistry were able when properly stimulated to accomplish in a single laboratory period by the individual method results as good as those secured by an equivalent group with identical experiments in a double laboratory period. Moreover, in planning laboratory exercises for a single period it is well to remember that it is no longer believed to be either necessary or desirable to have the pupils perform every exercise individually. Only enough individual experiments need be provided to make certain that every pupil secures the unique values to be had from such experiences. It is not necessary, therefore, to attempt to retain for that method the most difficult and time-consuming exercises. These, together with the more dangerous ones and those requiring a skill or delicacy of manipulation beyond what may reasonably be expected of a pupil may always be more profitably performed by teacher demonstration. It becomes our problem then to revise the exercises which we wish performed by individual pupils into short units which may be completed by the average pupil in from twenty-five to thirty-five minutes. Already certain schools have progressed far enough along these lines to justify the assurance that the task can be accomplished. Such effort is salutary. Doubtless the challenge of providing shorter exercises will pave the way for the shelving of some of the traditional exercises and of the introduction in their places of newer and more profitable ones.

*As a result of the depression there has been in the planning of*

<sup>2</sup> Victor A. Sugar, "An Investigation of the Relative Merits of the Single and the Double Laboratory Period in Teaching High School Chemistry." (Unpublished Study) University of Michigan.

*programs of studies a marked reversion to the doctrine of formal discipline and the transfer of training.* It is always challenging to note to how much greater an extent the doctrine of formal discipline is accepted by leading educators in England, France, and Germany than by our own leaders. It is interesting to observe to how much greater an extent the disciplinary subjects such as Greek, Latin, mathematics and science are revered abroad, possibly because of their effectiveness as means for selecting the most intelligent pupils and almost certainly because of their assumed values as media for developing intellectual power. It is intriguing to speculate upon the extent to which our more liberal discounting of the disciplinary values of these subjects may be due to our educational history of the past forty years. England, France, and Germany have had no high school movement such as ours. Perhaps they, too, would have been in a more receptive mood to evaluate and to accept the research evidence of Thorndike, Judd, and others with respect to limited transfer values had they been faced as we have been with an influx into their schools of millions of pupils for whom the study of the conventional subjects obviously holds little if any value, disciplinary or otherwise. But this is mere speculation, seductive but probably unprofitable and certainly impractical. Our present interest is primarily in the problem, to what extent has the obvious reversion to the doctrine of formal discipline affected the teaching of science in schools, especially those of the secondary level.

The movement toward an increased emphasis upon those subjects which have long been credited with affording excellent training for the mind has no doubt strengthened the position of physics and chemistry. In the present crisis even in those schools which have reduced their high school curricula to the extent that they now include nothing but the so-called solid subjects, physics and chemistry or both have been retained along with Latin, English, history, algebra and geometry. Physics and chemistry doubtless owe their places among these favored surviving subjects to the rather dubious distinction that the average high school pupil finds their mastery difficult. But the present status of biology as a subject entitled to remain on the same grounds is scarcely so well recognized. Its obvious values as a medium for giving training in scientific thinking and as therefore entitled to respect even among the advocates of formal discipline is likely to be overlooked by these advocates

because biology fails to be a bugbear to many pupils who lack the mathematical ability demanded for real success in high school physics and chemistry. Nevertheless during the period of the depression biology has been strengthening its position despite assumed shortcomings as a disciplinary subject. It has suffered practically no elimination even in schools which have most rigorously reduced their programs of studies. But general science has suffered to some extent in such schools. In the minds of some administrators and school board members general science is one of the "frills," and in a small percentage of cases it has been eliminated from junior high schools or from the ninth grade of four-year high schools on the grounds that, like music, manual arts and other victims of popular hysteria it is believed to possess chiefly cultural rather than disciplinary values.

The responsibility for nullifying this unfavorable reaction toward general science rests with teachers of science. We need first to re-evaluate in our minds the claims for formal discipline and to reassure ourselves that despite the present popular clamor for the elimination of all but difficult subjects which have enjoyed centuries of prestige, values will probably transfer from one school subject to others or to conditions of daily life only to the extent that there are common elements among the subject and the other subjects or the life conditions. We need to relegate formal discipline to the minor place it deserves among the aims of science teaching. We need then to refresh our memories concerning the mass of research evidence establishing the values of the general science course and then to defend general science for the values it undoubtedly possesses.

*An inevitable effect of the depression has been the encouragement of the use of makeshift laboratory apparatus and equipment.* The average board member or even administrator, to say nothing of the average tax payer has a very nebulous idea of what constitutes laboratory apparatus and equipment. To be convinced of this fact one has only to visit the room designated by the often flattering title of "laboratory" in any one of perhaps hundreds of small-town high schools and there to see the home-made laboratory tables, mute but eloquent evidence of the creative art of the school janitor or of the village carpenter, or there to examine the contents of the apparatus shelves and cases. There is too often the attitude on the part of the school officials that if the school possesses apparatus and equipment having the names and general appearances of the pieces desired, then

the school is adequately equipped. It is difficult to make any but science teachers comprehend why the clumsy laboratory tables are not "good enough" even though they lack hot water, gas, electricity, and all other conveniences and appurtenances that make manipulation and demonstration pleasurable and profitable. It is perhaps still more difficult to effect an understanding on the part of those responsible for administering the schools economically that the types of apparatus which are sometimes purchased under the demands for stringent economy may later prove to be expensive luxuries. There are facts about the purchase of laboratory supplies which those who place the orders should know. They should be informed that there are certain companies which through the years have built up reputations for producing laboratory apparatus of a high order of value and efficiency; that the products of these firms are not cheap in the sense that they can be priced extremely low; but that their products satisfy the demands for economical purchase because they will last well and will satisfy requirements meanwhile; that there are other firms selling apparatus which are not hampered by the limitations of maintaining a high standard of product. As one goes from school to school one all too frequently finds evidence that those who have purchased the laboratory supplies have not known these facts. One finds balances incapable of adjustment, cheaply and fragilely constructed models, incorrectly designed instruments, and unserviceable glassware. It is not unusual to see such ineffective and all but worthless apparatus lacking the stamp of any maker or distributor. Of course apparatus of this sort can be sold for much less than the prices which must be charged for standard high grade equipment. It is therefore easy during these days of limited school funds for representatives of firms marketing inferior apparatus to underbid more reputable firms and thus to sell their products to harassed administrators who are coerced by the necessity for making every cent go as far as possible.

The responsibility for discouraging the purchase of low priced but unserviceable apparatus and equipment must be shared jointly by the teachers of science and by the agencies which accredit secondary schools. We need to educate those who disburse school funds to the realization that the practice of submitting lists of apparatus and equipment to all distributors with the expectation that the order will go to the lowest bidder irrespective of the quality of his products is quite as illogical and may

prove quite as uneconomical as would the similar practice if applied to the purchase of food or other necessities for one's home. We need to educate administrators, school clerks, and board members to the fact that there is a vast difference in the potential effectiveness of various models and types of apparatus bearing the same name. Science teachers must be prepared to defend their requests for apparatus by explaining to the purchasing agent why the particular type or model specified will serve the intended purpose better than another. The responsibility of the inspector for the accrediting agency begins whenever he finds a school equipped inadequately or with apparatus ineffective for science teaching. He should make the immediate purchase of new equipment one of his recommendations for continued accrediting.

*An effect of the depression has been the placing of more and more responsibility upon the teachers of science for providing needed apparatus and equipment.* In many schools an increasing pressure is being brought to bear upon the teacher of science to improvise apparatus. It is not unusual to find a conscientious and capable teacher who devotes practically all the time which he has free from class and school routine, including evenings, Saturdays and even Sundays, to the making and repairing of equipment. While it is reasonable to expect the science teacher to make classroom use of many simple articles and to make simple repairs of apparatus, it is entirely unreasonable and unfair to expect him to devote any considerable share of his free time to this work. It should be brought to the attention of administrators that true economy in administration is measured less in dollars and cents than in the effectiveness of instruction secured; and that it is therefore far better to provide the science teachers with ready-made equipment and thus to insure to them sufficient time and energy to plan effective instructional activities than to force them to use a disproportionate share of their extra-class hours in the difficult and often discouraging task of improvising equipment.

*As a result of the depression there has been an increasing difficulty in effecting the purchase of necessary instructional materials.* Both the parents of the pupils and the school authorities who must conserve the meagre funds available have developed a growing resistance to the purchase of new books. Moreover school officials have found it imperative to curtail if not to eliminate entirely the use of mimeographed outlines and tests. As

a result of these economy measures many schools are no longer provided with the minimum equipment of necessary instructional materials. Administrators and teachers have attempted in several curious ways to meet the serious emergency brought about by the shortage of teaching materials. In some schools the pupils are forced to continue the use of textbooks which have become mutilated and tattered to the extent that they no longer serve as effective media of instruction; in others, antiquated textbooks which had been discarded years before but which unfortunately had remained stored on the school premises have been brought out and again put into active service; in other schools the classes have been provided with a weird collection of miscellaneous textbooks contributed by former pupils and by public-spirited citizens of the community; in still others teachers have had text-book purchases restricted to a point where the number of available texts is only a small fraction of the number of pupils in their classes. One such case recently reported was that of a teacher who possessed only seven books for her class of twenty-five. One could sympathize with her despairing statement that she had been unable to devise any plan by which she could insure that all the pupils could have access to these texts. She had unwillingly and as she averred ineffectively resorted to a lecture method of teaching.

The first step toward a reform of these conditions consists in educating the average citizen to realize that first class instruction is impossible under such conditions. So long as the average tax payer labors under the misapprehension that a way can somehow be found to provide adequate instruction under impossible handicaps, just so long will such conditions obtain. As soon as he becomes thoroughly convinced that he cannot have first class schooling for his children without paying for it, he will then be in a receptive mood to consider means of remedying the situation. The second step consists in making the average citizen aware of the fact that the serious curtailing of instructional equipment of all sorts is unnecessary even in a time of extreme financial depression. He needs to have pointed out to him the patent absurdity of curtailing educational facilities on the grounds that there is no money to provide them. He needs to be made to realize that his failure to finance the schools adequately constitutes a reflection upon his ability to appreciate relative values. Let us help the citizen to see what he has been spending his money for—in other words what he has been con-

sidering of more importance than maintaining high standards of education for his children. Here are some interesting facts for his consideration: According to the latest available authority the total annual cost of public education is approximately two billion three hundred million dollars. According to the recently published report of Ex-President Hoover's Committee on Recent Social Trends, the annual expenditure for commercial amusements, that is, moving pictures, drama, cabarets, night clubs, and radio broadcasting is approximately two billion two hundred million dollars, a sum about equal to that spent for public education; that for travel by American citizens is approximately six and one half billion dollars, or nearly three times the entire cost of public education; that for games, sports and outdoor life, nearly a billion dollars; and that for all forms of recreation, including those previously mentioned, more than ten billion dollars annually or about five times the cost of all public education. A further interesting item of information for general dissemination is the fact that the American people spend annually in the "pursuit of personal beauty" about a billion dollars or an amount equal to nearly half the total sum expended for public education. Exact data concerning the annual sums spent on tobacco are not available, but a study of the quantities of cigarettes, cigars, and other forms of tobacco consumed indicates that the American people are willing to spend for these commodities many times what they have been spending for the public education of their children. These are only a few items of data the knowledge of which may become a potent means of bringing the people to realize that a sober consideration of the relative values of the items for which they are spending their money is in order. From such facts it should be readily apparent that all the money necessary for maintaining and improving our educational program is available provided our people are willing to forego some of their luxuries in order to uphold the schools.

*As a result of the depression there has been an increasing tendency toward the use of some form of unit plan.* Many though probably by no means a majority of experts believe that unit teaching is the best means of individualizing instruction in any class. Perhaps a greater number of teachers and supervisors believe some form of unit technique to be if not the only practicable plan at least the most practicable one with large classes. The acceptance of this belief, however, should be based upon a fair

appraisal of the merits and demerits of unit techniques and not upon the relative ease of their administration. With many teachers of science, developmental methods employing skillful oral questioning and direction are likely to be relegated to a minor position or to be eliminated entirely in the more strictly individual work of any form of contract, project, or unit plan. We should be slow to discard without deliberate consideration any teaching techniques which have long proved effective, even though their application be difficult in a changed plan of work. Here and there gifted teachers are learning the difficult task of conducting developmental and socialized procedures with classes of fifty or more. Others are successfully introducing such activities into large classes by carrying on varied activities with different groups within the same class. The situation calls for much intelligent trial and error teaching in the never-ending search for means toward more effective instruction. Out of this necessity are certain to emerge new skills and new devices which will be added as permanent contributions to the art and the science of teaching.

*The depression is responsible for a lowering of standards for teachers of science.* With thousands of qualified teachers of science available it would logically be expected that the intense competition for positions would place a premium upon the possession of superior qualifications. Paradoxical as it may seem the results of the depression are in sharp contrast with these expectations. Neither the economic "law of supply and demand" nor the biological law of "survival of the fittest" has held. To be sure, intelligent administrators in many school systems have made the most of the oversupply of highly qualified, competent and experienced teachers to raise the levels of their corps. They have seized the opportunity to get rid of the weak and the lazy members of their staffs and to put in their places teachers of proved ability and worth from the ranks of unemployed. If every school system had been pursuing this policy during the last four years, we would be justified in saying that the depression had resulted in at least one outstanding reform. We should be justified in feeling that great progress had been made toward the ultimate establishment of a teaching profession having the prestige now held by law or medicine.

But in an astonishing number of schools capable teachers have been replaced with wholly incapable ones. Men and women who had been long engaged in other callings eagerly turned to

teaching as the only possibility of employment when because of the depression their chosen callings no longer offered them a livelihood. Blessing the prudent foresight which at the time of their graduation from college many years before had prompted them to secure a teachers' certificate as a sort of insurance against unemployment in the event of a subsequent serious crisis, they rescued the precious, long forgotten certificate from the bottom of the trunk, blew the dust from it and presented themselves as qualified applicants. Some of these overnight educators secured positions through the fortunate accident of relationship to important board members or taxpayers; others secured appointment by underbidding the qualified teachers who had prepared for teaching with the intention and hope of making it a life work; still others gained positions because of the increasing provincialism which has been one of the astonishing and disturbing phenomena of the depression, that is, many have secured employment as teachers whose chief qualification has been the fortuitous accident by which they happened to have been born and raised in the district.

Tragically enough relatively few citizens except those who have a real understanding of what constitutes equipment for expert teaching have been especially concerned over this mad rush of untrained and undertrained into the ranks of teachers. The average individual is too prone to believe himself a thoroughly qualified authority on all educational problems and to believe that the possession of a modicum of education of almost any sort constitutes adequate qualification for teaching. It is probable and is surely "a consummation devoutly to be wished" that many of the recent undesirable additions to the ranks of teachers will be reabsorbed by industry as economic conditions improve. But incalculable harm has been done by even their temporary admission to the calling of teaching.

Of the sciences perhaps general science and biology have suffered most from this influx of incompetent teachers into our schools. But unfortunately some administrators share the opinion expressed some years ago by the superintendent in a city of nearly a quarter of a million inhabitants, who stated that anybody can teach general science and who expressed cynical surprise that there should be any one so fatuous as to challenge his profound pronouncement. But not all the harm done to the cause of science teaching has resulted from its assignment to teachers with inadequate subject-matter training. In many

schools industrial chemists have been appointed as chemistry teachers and engineers of all sorts have been assigned the task of teaching physics. The highly technical knowledge which such specialists possess unless it is accompanied by a thorough understanding of adolescent psychology and a functional knowledge of the theory and practice of teaching is not unlikely to result in presentations entirely unsuited to the comprehension of the pupils.

What can be done in the face of such a situation as these incidents reveal? How can we lay plans now to prevent a recurrence should a similar economic crisis occur a decade or so hence? Exhaustive research investigations point out the practical certainty that henceforth there will always be an abundance if not an oversupply of teachers. How then can we secure and insure a profession consisting of only those best qualified to belong? The first step must be toward establishing in the minds of the average citizen a greater respect for the teaching calling. To attain this end requires a carefully planned program of publicity and education. Through the press and through influential magazines in sympathy with the cause of public education for all children, there need to be series of articles and editorials giving facts about education with emphasis upon its importance and its abstruse and multiform problems; and about the nature and extent of the training which our foremost experts believe necessary before a candidate may become a qualified teacher. The average citizen needs to be informed that there is such a thing as applied psychology and that there is at least an art if not as yet a science of teaching. He needs to know in terms of dollars and of years of training the cost of the investment one must make in order to attain the qualifications of the skilled and competent teacher. It will astonish many a business man to learn that the science teacher in the local high school has invested in education more money than he himself has invested in his business. Wide publicity must be given to the fact that thoroughly qualified teachers must be and are a very highly selected group—highly selected upon the basis of intelligence, personality, industry and many other traits. As classroom teachers we need more than ever before to strive to improve our own efficiency in order that the contrast between really expert teaching and inexpert lesson hearing will be more clearly manifest to even the untrained observer.

## CONTRIBUTIONS OF THE PERIOD 1450-1650 TO THE SUBJECT MATTER OF HIGH SCHOOL MATHEMATICS

BY GLADYS V. WORDEN

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This article has the following purposes: to study the period 1450 to 1650 in mathematical history in order that the contributions of this period to the subject matter of high school mathematics may be determined, to show by this study how slowly new ideas were universally adopted, how each man had only his own ingenuity and what had been discovered before him with which to work, and how one man's ideas were gradually developed by men succeeding him. It is also hoped that the reader may more fully appreciate mathematics as it is now taught, and realize through reading anecdotes concerning these mathematicians of long ago that they were human after all as we are.

The period 1450 to 1650 has been chosen for intensive study of what was contributed to the subject matter of high school mathematics because there is no other period of equal length so rich in mathematical discoveries in subject matter of the high school level. After this period most contributions were in the field of college and advanced mathematics. Before this period progress was much slower. Two of the three most important mathematical inventions appeared during these 200 years, namely decimal fractions and logarithms. The miraculous powers of modern calculation (6) are due to these two inventions and to the system of Hindu Arabic notation in use long before 1450.

The subjects included in high school mathematics are those commonly taught in the junior and senior high schools of New York State: arithmetic (elementary and commercial), algebra (elementary, intermediate, and advanced), geometry (plane and solid), and trigonometry. The topics included in the subject matter of each are those listed in current Regents' syllabi.

Chronological order will be maintained so far as possible that the development of mathematical discoveries may better be followed. We may now proceed to the first important mathematician of this period.

Since printing was invented about the middle of the fifteenth

century, we might expect textbooks to be printed in the years following. The first printed arithmetic (6)\* appeared at Treviso, Italy in 1478. The author is unknown. Five fundamental operations (23) were considered: numeration, addition, subtraction, multiplication, and division; i was used for the figure 1. A sample multiplication table is:

$$\begin{array}{l} i \text{ times } i \text{ makes } i \\ i \text{ times } 2 \text{ makes } 2, \text{ etc.} \end{array}$$

Addition problems were of this type:

$$\begin{array}{r} 59 \\ 38 \\ \hline \text{sum} & 97 \end{array}$$

Addition and subtraction were both proved by casting out 9's, as follows:

$$\begin{array}{r} .59 \\ .38 \\ \hline \text{sum} & .97 | 7 \end{array}$$

It is interesting to note a typical problem:—"Three merchants have invested their money in a partnership, whom to make the problem clearer I will mention by name. The first was called Piero, the second Polo, and the third Zuanne. Piero put in 112 ducats, Polo 200 ducats, and Zuanne 142 ducats. At the end of a certain period they found that they had gained 563 ducats. Required to know how much falls to each man so that no one shall be cheated." (23:12)

The first printed German arithmetic (6) came out in 1482. Only fragments of the first copy are now extant. The first printed treatise to contain the word "zero" appeared at Florence in 1491. Luca Pacioli (1445-1514?), a Tuscan monk, in 1494 printed at Venice three books introducing several symbols in algebra. His (24) was the first general work on mathematics, and it included algebra, arithmetic, and trigonometry. Addition and subtraction were indicated by p. and m. respectively. One of Pacioli's problems reads thus, "A mouse is at the top of a poplar tree that is 60 feet high, and a cat is on the ground at its foot. The mouse descends half a foot each day and at night it turns back  $1/6$  of a foot. The cat climbs one foot a day and

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\* See bibliography.

goes back  $\frac{1}{4}$  of a foot each night. The tree grows  $\frac{1}{4}$  of a foot between the cat and the mouse each day, and it shrinks  $\frac{1}{8}$  of a foot every night. In how many days will the cat reach the mouse, and how many ells has the tree grown in the meantime, and how far does the cat climb?" (18:207, 208)

The arithmetic (6) of John Widmann (1460–?) published in 1489 in Leipzig was the earliest printed book (5) in which + and – are found, used not as symbols of operation (20) but to express an excess or deficiency in packages of merchandise. He (6) used + for "et" and "and." The word "plus" does not occur in his text; the word "minus" does two or three times. Our plus sign comes from the Latin "et" as it was written in manuscripts before the invention of printing. The origin of minus sign is still uncertain. Universal adoption of these signs was slow.

The first complete solution (11) of the equation  $x^3 + mx = n$  was worked out by Scipione del Ferro (1465–1526) who was an Italian, but his solution has been lost. The second solution of this equation was the work of Tartaglia of whom we shall speak later. Ferro (6) was a professor of mathematics at Bologna. He told his pupil, Floridas, of this solution in 1505, but did not publish it. At that time discoveries were kept secret in order that an advantage might be gained over rivals. This practice led to many disputes as to the priority of inventions as we shall see later.

In 1522 the first important English arithmetic (6) was published in Latin by Cuthbert Tonstall (1474–1559). He (20) began his study of mathematics as a result of trying to check up on accounts with goldsmiths whom he suspected of inaccuracies.

An interesting character of this time was Michael Stifel (1486?–1567), a German (20), who had been trained in a monastery. He was attracted by Luther, and became a religious fanatic. One day he started for heaven with a group of peasants, but he ended up in jail. Luther got him released, but Stifel still nursed his grievance against Leo X who was then Pope. He had been interested in studying the significance of mystic numbers in the books of Daniel and Revelation. At last he proved what he wished to by following certain steps in reasoning.

1. Latin for Leo X is Leo Decimus.
2. This may be written Leo DECIMVS.
3. The capitals may be arranged thus: MDCLVI.
4. Take away M for mystery, and add X for Leo X and get DCLXVI.

5. This equals 666, or the number of the beast in Revelation. Therefore, Leo is the beast.

His interest in mystic numbers led him to a study of mathematics, and he (6) became the greatest German algebraist of the sixteenth century. In 1544 he published "Arithmetica Integra" in Latin in three parts:—rational numbers, irrational numbers, and algebra. He gave a table of numerical values of binomial coefficients for powers below the 18th. He saw an advantage in letting a geometric progression correspond to an arithmetic progression. This was the beginning of the theory of exponents and logarithms. Until the seventeenth century mathematics dealt exclusively with absolute positive quantities, the conception of negative quantities being slow. Stifel said, "Negative numbers are less than nothing." (5:233)

Another textbook (20) appeared in Florence in 1491, an arithmetic published by Filippo Calandre containing the first printed example of modern long division, and illustrated problems.

We now come to Tartaglia who has already been mentioned in connection with the solving of  $x^3 + mx = n$ . His real name (6) was Nicolo of Brescia (1499–1557). When he was six years old his father (2) was killed by French soldiers, who also split Tartaglia's skull in three places and cut his jaws and palate. His mother managed to keep him alive, but he always stammered after that, and was called Tartaglia (6) meaning stammerer. His family was poor, and he (2) attended school only 15 days during which time he stole a copy book, and taught himself to read and write. He (6) also learned Latin, Greek and mathematics by himself. He (2) could not afford to buy paper; so he used a tombstone as a slate on which to work his exercises.

In February 1535 Floridas and Tartaglia (6) held a public discussion, each proposing 30 problems. The one to solve the greatest number in 50 days was to be victor. Tartaglia solved 30 of the problems of Floridas in two hours; Floridas solved none of Tartaglia's. In 1541 Tartaglia discovered the general solution for  $x^3 \pm px^2 = \pm q$  by transforming it into the form  $x^3 \pm mx = \pm n$ . He refused to publish his method. In 1539 Hieronimo Cardano (1501–1576), commonly called Cardan (11), secured the solution under pledge of secrecy. He swore by the honor of a gentleman not to publish Tartaglia's inventions. However, he published the solution in 1545 in his book "Ars Magna," but he did mention Tartaglia's name in connection with it. Unable to forget that

Cardan had broken his promises, Tartaglia (6) challenged Cardan and his pupil, Lodovico Ferrari (1522–1565), to a contest. They were to solve 31 problems in 15 days. Tartaglia solved most of them in 7 days; the others did not send in solutions until the end of 5 months, and then all but one were incorrect. Tartaglia died before publishing his work.

Cardan (6) called the negative roots of an equation fictitious, and the positive roots real. He failed to recognize imaginary roots. He wrote  $(5 + \sqrt{-15})(5 - \sqrt{-15})$  as  $5.\tilde{m} R/. \tilde{m}. 15$

$$\frac{5.\tilde{m}. R/. \tilde{m}. 15}{25 \tilde{m} \tilde{m} 15. \text{quod est } 40}$$

He (20) has been described as a genius with no principles. In a fit of anger one day he (2) cut off the ears of his younger son who had done something to displease him. He was interested in astrology, and received a pension for serving as astrologer to the court of the Pope. He foretold his own death on a certain day, and in order to keep up his reputation, he killed himself on that day. In 1663 there (6) was published posthumously his gambler's manual "De Ludo Aleae" discussing the theory of probability. Thus, we leave Cardan, the man whose character we can not admire, but whose mathematics we can.

Robert Recorde (1510–1558) was the author (6) of the first English treatise on algebra, "The Whetstone of Witte," published in 1557. The title page, among other things, has the following:

Though many stones do beare greate price  
The Whetstone is for exercise

The Grounde of Artes did brede this stone:  
His use is greate, and more then one.  
Here if you list your wittes to whette,  
Moche sharpnesse thereby shall you gette.  
Dull wittes are fined to their fulle end.  
Now prove, and praise, as you doe finde,  
And to yourself be not unkinde. (8:43)

He (6) was the first to use the equality sign. He selected it because no two things could be more equal than two parallel lines. The division sign was first used by Johann Heinrich Rahn, a Swiss, in his "Teutsche Algebra" published at Zurich in 1659.

Perhaps we do not appreciate the work that has been required to make tables which we use and take for granted. A German, Georg Joachim (6), better known as Rhaeticus (1514–1567), cal-

culated a table of sines with a radius equal to 10,000,000,000, and later with one equal to 1,000,000,000,000,000. He began the tables of tangents and secants, but died before finishing them. He employed several calculators for 12 years. His work was completed by Valentine Otho (1550?-1605), also a German. Later Bartholomaeus Pitiscus (1561-1613) freed these tables from errors, and republished them.

Rudolff (23) in 1530 used the symbol | as a decimal point in compound interest tables. In 1530 he (18) added fractions of unlike denominators by putting the common denominator below them, and the new numerator above them, thus:

$$\begin{array}{r} 8 & 9 \\ \hline 2/3 & 3/4 \\ \hline 12 \end{array} \text{ makes } 17/12$$

Francis Vieta (1540-1603), sometimes called the father of modern algebra (17), published a work (6) in 1600 introducing ordinary root extraction. In 1579 he published a book contributing to trigonometry, it being the first systematic elaboration of methods of computing plane and spherical triangles by the six trigonometric functions. In equations he rejected all but positive roots. He (17) was one of the first to use letters in algebra to denote general or indefinite quantities. The equation  $a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$  was written by him, "a cubus +  $b$  in a quadr.  $3+a$  in  $b$  quad.  $3+b$  cubo aequalia  $a+b$  cubo." The equals sign was not then in universal use. Quadratus (4) meant square; cubus meant cube. He (6) used  $N$  to represent an unknown quantity,  $Q$  to represent its square,  $C$  to represent its cube.  $x^3 + 8x^2 + 16x = 40$  was written  $1C + 8Q + 16N$  aequal 40. He used the terms "coefficient" and "polynomial," the Maltese cross (+) for addition, and - for subtraction. To Vieta goes the honor for being the first to get an explicit expression for  $\pi$ . He (17) also represented the product by an area when each factor represents a length. Vieta (11) went farther than Cardan in solving equations, and discovered a method for solving equations of any degree. It (20) has been said that he studied mathematics for the fun of it in his leisure hours.

Ludolph van Ceulen (1540-1610) of the Netherlands (6) aided by Adrianus Romanus (1561-1615) approximated the ratio of a circle to its circumference. Romanus carried the value to 15 places; Van Ceulen carried it to 35 places. The quantity  $\pi$

was often known as "Ludolph's number," and his performance was considered so extraordinary that the numbers were cut in his tombstone, now lost, in St. Peter's churchyard at Leyden.

The importance of the invention of decimal fractions has already been mentioned. The honor of inventing them goes to Simon Stevin (1548–1620) of Bruges, Belgium. He (6) was the first to use them systematically. In a book in 1585 he described the advantages of decimal fractions and of decimal division in systems of weights and measures. In place of a decimal point he used a cipher. To each place in the fraction was attached the corresponding index. No improvement was made in decimals until the beginning of the 17th century. A typical multiplication of decimals (23) appeared thus:

$$\begin{array}{r}
 4 \ 5 \ 6^* \\
 3 \ 7 \ 8 \\
 \hline
 5 \ 4 \ 2 \\
 \hline
 1 \ 5 \ 1 \ 2 \\
 1 \ 8 \ 9 \ 0 \\
 \hline
 2 \ 0 \ 4 \ 1 \ 2 \\
 4 \ 5 \ 6 \ 7 \ 8
 \end{array}$$

In 1585 Stevin (24) used  $\sqrt{3}$ ,  $\sqrt[4]{4}$ , etc. for cube root, fourth root, etc. He (18) indicated  $\sqrt{+2\sqrt{3}}$  by 1 bino  $2+\sqrt{3}$ . He also used fractional exponents, and wrote  $a^3 - 2a^2 + 3a - 4$  as  $3 - 22 + 31 - 40$ . There (6) is doubt about who used the first decimal point. Probably it was John Napier (1550–1617) of Scotland whom we honor as the inventor of logarithms.

Napier (5) was sent abroad to school because his uncle once wrote to his father, "'I pray you, Sir, to send John to the schools either of France or Flanders, for he can learn no good at home.'" (5:155). In 1574 a beautiful castle was built for him on the banks of the Endrick. On the opposite side of the river was a lint mill, and the noise from it disturbed Napier so that he wanted the miller to stop it that his train of thought might not be broken. He (18) was hereditary poultry keeper for the king, and had a black rooster. When he suspected that one of his servants had cheated him, he said that the rooster would identify the guilty one. He coated the rooster with lampblack, put him in a darkened room, and told the servants to go in separately and pet him. The guilty servant was afraid to touch the rooster, and came out with clean hands. Therefore, Napier won the reputation of being a magician.

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\* The digits printed in bold face type were set inside small circles in the original.

Unusual as it may seem, it is nevertheless true that Napier constructed logarithms before exponents were used. His logarithms were not the same as natural logarithms. He took the logarithm of  $10^7 = 0$ . The idea of a "base" is inapplicable to his system. He published his table of logarithms in 1614, and (13) called them not logarithms, but "numeri artificiales." He (18) wrote 3.14 as 3,14 or 3,1'4".

Henry Briggs (1556–1631) of London (6) suggested logarithms based on 10, but it (13) has been said that Napier had thought of this before Briggs suggested it to him. Briggs (6) published in 1624 his "Arithmetica Logarithmica" containing logarithms to 14 places of the numbers 1 to 20,000, and 90,000 to 100,000. The gap was filled later by Adrian Vlacq (1600?–1667?). The word "characteristic" first occurs in Briggs's book; "mantissa" did not come into use until 1693. Others who later corrected and added to Napier's work were Joost Burgi (1552–1632), Johannes Kepler (1571–1630), and John Speidell.

Another mathematician of this period was William Oughtred (1574–1660), an Englishman who used to study all night sometimes, and teach free those interested in mathematics. He used altogether over 150 symbols. Three that have remained are  $\times$  as a symbol of multiplication,  $::$  as that of proportion, and  $\sim$  as a symbol for difference. Oughtred did not use parentheses, but enclosed between double colons terms to be aggregated, thus:— $\sqrt{(A+E)} = \sqrt{q:A+E::}$ . Parentheses were used earlier, but did not become popular before the time of Leibnitz and Bernoulli. Oughtred invented the circular and rectilinear slide rules about 1621.

It (6) has been said that Oughtred's wife was so economical that she denied him the use of a candle for study in the evening.

A mathematics teacher in London by the name of Richard Delemain (23) published a pamphlet in 1630 describing the circular slide rule which he probably invented independently of Oughtred.

Edmund Gunter (1581–1626) of London (6) invented the words "cosine" and "cotangent" in 1620. Cosine was the abbreviation for complementary sine. "Tangent" and "secant" were invented by a physician, Thomas Finck of Basel in 1583. Gunter (18) wished to be professor of geometry at Oxford. He appeared before Sir Henry Savile, brought his instruments, and "fell to resolving of triangles and doing a great many fine things. Said the grave knight, 'Doe you call this reading of Geometrie? This

is showing of tricks, man!" and so dismissed him with scorn, and sent for Henry Briggs of Cambridge." (18:343)

René Descartes (1596–1650) of France (6) destroyed old ideas, and built up new ones. He is described as being "a small man with large head, projecting brow, prominent nose, and black hair coming down to his eyebrows. His voice was feeble. In disposition he was cold and selfish." (2:280) He (12) was introspective and quite morbid, very much interested in philosophy, and a gentleman who had been trained in a Jesuit school, and who never forgot the courtesies and conventions of society. He (6) used equations of more than one unknown so that in case of two unknowns for any value of one unknown (abscissa), the length of the other unknown (ordinate) could be computed. The letters  $x$  and  $y$  were used for abscissa and ordinate. The words "abscissa" and "ordinate" were not used by Descartes. The Cartesian system of coordinates (17) was named after Descartes whose Latin name was Cartesius. Descartes represented the product of  $a$  and  $b$  where  $a$  and  $b$  correspond to the lengths  $L$  and  $L'$  by a length which has the same ratio to  $L$  as  $L'$  has to the unit length. He found that the polynomial  $f(x)$  is divisible by  $x-a$  whenever  $a$  is a zero of this polynomial. He (24) used  $\infty$  and  $\infty$  for  $ae$  in aequalis, and he first used the word "function" in 1637 for an integral power of a variable  $x$  as  $x^3$ ,  $x^2$ , etc. Instead of the "Acubus" of Vieta (6) and  $aaaa$  for  $a^4$  of Harriot, Descartes used  $a^4$ , etc. The idea spread quickly. Probably Descartes is best known to the average student for his rule of signs which states that the number of positive roots can not exceed the number of variations in the signs of the coefficients, and the number of negative roots can not exceed the number of permanences of sign. He has been criticized for not mentioning imaginary roots, but in the original French he said that the equation "may have" so many roots, not "always has."

In 1649 he accepted the invitation of Queen Christina to the Swedish Court, and he died in Stockholm in 1650.

Pierre de Fermat (1601–1665) and Blaise Pascal (1623–1662), both of France, founded the theory of probability. Fundamental in the development of this theory was correspondence which took place between Fermat and Pascal and a gambler, Chevalier de Méré. Pascal at the age of 12 had a genius for mathematics. His father forbade him to talk about it, but urged him to learn Latin and Greek. The boy drew figures with charcoal on the pavement, and unaided discovered that the sum of the three

angles of a triangle is two right angles. His father wept with joy when he found it out. Pascal (6) died from overwork when he was 39 years old.

Other men of this period who have not been mentioned here for lack of space are:—John Mueller (1436–1476), Leonardo da Vinci (1452–1519), Johannes Werner (1468–1528), Nicholas Chuquet, Pellos, Franciscus Maurolycus (1494–1575), Jacques Peletier (1517–1582), Adam Reise, Raphael Bombelli (1530–1560), Thomas Harriot (1560–1621), Christopher Clavius, Albert Girard (1590?–1633?). Their contributions were varied and helped to fill the gaps in the slowly developing mathematical ideas.

As can be seen, people were slow to adopt new mathematical ideas even after they had been known for many years, and many times we do not appreciate the labor that was involved in the discovery of those ideas. G. A. Miller (17) says that although the decimal system was introduced into Europe in the sixteenth century, yet we are still using other systems. For example, we use the base 60 when we divide an hour into 60 minutes; and we still use the unwieldy English system of weights and measures. The Roman system is yet used for some things. We persist because we are unpractical. Perhaps, then, we should not too freely criticize those people of old who so slowly adapted themselves to progress in the realm of mathematics.

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**TERRARIA AND THEIR PLANTS**

BY EDWIN D. HULL

*Hull Botanical House, Gary, Indiana***GENERAL**

The primary purpose of terraria is to demonstrate ecological groups. Secondarily they provide material for the study of vegetative parts of plants, and to a lesser degree they are helpful in providing flowers. It would be impossible to grow most of these plants otherwise except in expensive greenhouses, as they are much too delicate for the most part to withstand the severe conditions of the schoolroom.

The usual terrarium is an aquarium, either round, rectangular, or with two sides flattened, and covered with a glass plate. Such a contrivance does not allow for drainage, and permits little if any circulation of air. The result is stagnation, and, with one exception, that of the swamp, this is detrimental to the plants, even such plants as are found in bogs, which decay readily under such a condition. Better equipment can be had. A fern pan, placed under a bell jar which is slightly raised at the bottom, is good, and the entire glass can be easily removed for brief periods for better purposes of study. A propagating box described by Mary Louisa Hellings under the title "Grow Holly," Nature Magazine, December, 1932, is also good. Best of all is a terrarium with a base of wood, bored to allow drainage, as in a window-box, and with sides and top of glass, although in certain cases the top will not be needed, as will be noted later. The glass should not fit so tight as to be air-tight. Such a terrarium can be made by a local carpenter, and will be found neat and serviceable. If an ordinary aquarium is used the soil for the plants should rest on a thick layer of absorbent material, such as sand or sphagnum moss. Some charcoal added to the soil will help to keep it sweet.

Each ecological group will require separate treatment, and it is imprudent to mix plants of different groups in the same receptacle, as for instance a pitcher-plant with a hepatica.

Soil is especially important, and it is to a neglect of this factor that most failures are probably due. Very few of the plants will grow in ordinary garden soil. It is best to use soil from places where the plants are found.

Many plants will be benefited if in summer the terraria are

placed out-of-doors in partial shade regardless of light requirements indoors. Plants of sunny arid situations, however, are best kept in full sun the year round.

All species are best planted in autumn.

Many plants will flower seldom if at all, especially the training evergreens, but these are desirable for their foliage and other vegetative organs, and usually they will live long and will reproduce vegetatively under artificial conditions. Rosette and bulbous plants and the like will usually bloom, at least for the first year, and bloom far ahead of their natural flowering time.

It is not advisable to keep animals where plants are desired, as they will by burrowing, etc. uproot the plants, and cause general destruction.

On an ecological basis terraria may be classified as follows:

**Bog Terrarium:** Requirements for this terrarium are a highly acid soil, with abundant moisture both in soil and air, and in winter as much sunlight as possible. Little sunlight will cause an abnormal growth of the leaves of some insectivorous plants, as for example, the Venus Fly-Trap, which will run mostly to petiole, and the blade, which is the trap part, will develop scarcely at all. This is perhaps the most interesting of all terraria, containing all the terrestrial insectivorous plants, and a number of other forms of considerable botanical value. The insectivorous plants include the Sundews, Pitcher Plants, Venus Fly-Trap, and the Butterworts. Of the Sundews a round-leaved species, *Drosera rotundifolia*, seems to be the best. It is common in the north, but it is best to obtain specimens from the south, as these are in a leafy condition much longer. The Sundews pass into a resting stage in winter, existing above ground only as buds, but will resume growth early in the year. Of the Pitcher Plants *Sarracenia purpurea*, of northern and southern distribution, is the one most commonly grown, but here again specimens from the south seem to be superior. A species popularly known as "Trumpets" (*Sarracenia flava*), a southern plant, is excellent, flowering year after year indoors. Being a large form, however, it is suitable for only large terraria. The California Pitcher Plant (*Darlingtonia californica*) can now be had from some dealers and is a very curious and beautiful plant, with an elaborate mechanism for attracting insects. This, too, is a large species. The Venus Fly-Trap (*Dionaea muscipula*) has long been cultivated and should live for many years. The Butterworts (*Pinguicula*) have not heretofore been known as terrarium

plants. They are all of small size. One species is found in the far north, while a few forms occur in the south. Of these southern plants *Pinguicula elatior* has been tried and found to be of very easy culture. The general habit, and the flower, which is purplish-white, are shown in the illustration. The leaves are fleshy and thickly covered with small glands, and have a buttery appearance. The plants show, perhaps, the insectivorous habit in its simplest form. During the flowering period (in late winter indoors) the plant passes into a resting stage with much smaller leaves. These leaves when carefully removed from the plant and placed on damp bog soil and kept covered with a glass plate give rise readily to new plants, from one to three plants from each leaf.

Among other flowering plants suitable for the bog terrarium are the common Cranberry of commerce (*Vaccinium macrocarpon*), which is long lived though it may not flower, and a few orchids, such as the Grass Pink (*Calopogon pulchellus*), and a small species of Twayblade (*Liparis Loeselii*). Of the lower forms the Bog Moss (*Sphagnum*), very characteristic of bogs, and the Liverwort (*Conocephalus conicus*), are to be recommended. *Sphagnum* should have a place in every bog terrarium, but a small bunch should be enough, as it is not at all necessary that other species be planted in it, and a large quantity would almost hide the smaller plants from view.

*Swamp Terrarium:* The receptacle in this case should be an aquarium. Requirements are a somewhat acid and wet soil, and abundant sunlight. It might be well to have one-half the soil built up above water, and the other half inundated to a depth of from one to two inches. This terrarium may not need a top. To give a natural effect the larger part of the plants should consist of sedges and rushes, small specimens of which are easily secured and should do well. Particularly recommended are the Spike Rush (*Eleocharis acicularis*), also a good aquarium plant, and small species of *Cyperus*, *Carex* and *Juncus*. The Umbrella Plant (*Cyperus umbellatus*), long known as a house plant, is exceedingly easy to grow. Small specimens of the Sensitive Fern (*Onoclea sensibilis*) are good, as are also the fern allies *Selaginella apus*, a creeping moss-like plant, the Four-leaved Water Clover (*Marsilea quadrifolia*), and the Quillwort (*Isoetes lacustris*), the latter also a good aquarium plant. The Water Pennywort (*Hydrocotyle umbellata*), a member of the Umbelliferae, grows so rapidly in cultivation that portions of it will probably have to

be removed from time to time. The Marsh Marigold (*Caltha palustris*) does well, and Ladies' Tresses (*Spiranthes cernua*), an orchid with fragrant white flowers, will also grow. *Onoclea*, *Selaginella*, *Hydrocotyle*, and *Spiranthes* are best planted above the water line, the others below it.

*Woodland Terrarium:* This is divided into two sections.

(a) *Evergreen (Coniferous) Woods:* This terrarium needs a light, sandy soil of a high degree of acidity, best mixed with



I. Left to right. Upper row: Water Pennywort (*Hydrocotyle umbellata*); Butterwort (*Pinguicula elatior*). Lower row: California Pitcher Plant (*Darlingtonia californica*); Sensitive Fern (*Onoclea sensibilis*).

the leaves of some conifer, such as a pine, partial shade and considerable moisture, though good drainage should be provided. A cool situation is best, since the plants are mainly northern in distribution. Here predominate members of the Heath Family (*Ericaceae*), among which can be recommended species of *Chimaphila*, Shin-Leaf (*Pyrola*), Wintergreen (*Gaultheria procumbens*), and the Trailing Arbutus (*Epigaea repens*), which has been successfully grown from seeds, though it should not be transplanted from the wild. The Rattlesnake Plantain (*Epi-pactis pubescens*), an orchid with dark green leaves veined with

white, is very attractive, likewise the Dwarf Cornel or Bunchberry (*Cornus canadensis*) the Harebell (*Campanula rotundifolia*), and the dainty orchid *Calypso bulbosa*. The Reindeer Lichen (*Cladonia rangiferina*) belongs here, as well as some large attractive mosses, such as the Pine-Tree Moss (*Climacium americanum*), a species of Hair-Cap Moss (*Polytrichum juniperinum*), and *Mnium*, the last a trailing species, and probably the most easily grown of all mosses.



II. Left to right. Upper row: Tuber of Tuberous Sword Fern (*Nephrolepis cordifolia*); Little Club Moss (*Selaginella apus*). Lower row: Edelweiss (*Leontopodium alpinum*); Juvenile House Holly Fern (*Cyrtomium falcatum*).

(b) *Deciduous Woods*: Requirements are a porous, somewhat acid soil with an abundance of leaf mold, considerable moisture and partial shade. Plants in this group may be regarded as typical mesophytes. A number of lichens can be grown, though these will not live unless the air is pure. The Liverworts *Conocephalus conicus*, also found in bogs, and *Marchantia polymorpha*, are easily grown, although the latter will probably do better if a little charred wood is added. Of the Mosses, *Mnium*, White Moss (*Leucobryum glaucum*), the Common Hair-Cap

Moss (*Polytrichum commune*), and the Purple-Stemmed Moss (*Ceratodon purpureum*), have been tried and found satisfactory. Many other species of moss would probably do equally as well. The Shining Club Moss (*Lycopodium lucidulum*), which is a fern ally and not a moss, is good. This species bears abundant gemmae, and these will grow slowly if detached from the plant and placed on the surface of the soil. The Little Club Moss (*Selaginella Kraussiana*), which likewise is not a moss, but belongs to the fern group, the most common green house species, does very well, and will probably have to be thinned out from time to time. Many native ferns naturally small, and small specimens of larger ferns, will do well. Small specimens of cultivated ferns, as the House Holly Fern (*Cyrtomium falcatum*), the Hart's Tongue Fern (*Scolopendrium vulgare*), and a species of Brake (*Pteris cretica*), are good. A species of Sword Fern (*Nephrolepis cordifolia*) bears abundant tubers underground, and much pleasure can be had in growing plants from them. From one to three plants may be expected from each tuber in about two months time. A Trout Lily (*Erythronium Hartwegii*) of the western U. S. and with yellow flowers is easy, likewise the Wild Lily of the Valley (*Maianthemum canadense*), Twayblade Orchid (*Liparis liliifolia*), Liverleaf (*Hepatica triloba*) Goldthread (*Coptis trifolia*), Star Flower (*Trientalis americana*), and the Partridge Berry (*Mitchella repens*). Species of *Trillium* will grow in ordinary garden soil, but these are mostly too large, though there are two small species well worth a trial, *Trillium nivale*, a white-flowered form of the middle west, and *Trillium rivale*, a mauve colored flower of the far west.

*Prairie Terrarium:* The prairie is intermediate between swamp and plains. Requirements are a well-drained, slightly acid soil, full sun and a moderate supply of moisture. A top may not be needed. The basis of this terrarium will be perennial grasses, of which there are numerous kinds both native and foreign. Native and naturalized grasses are easily obtained in most localities; if they grow too large they can be trimmed. Ornamental variegated grasses are offered for sale in many seed catalogues. Outside the grasses suitable plants do not seem to be numerous. Small species of clover, such as the White Clover (*Trifolium repens*) may be planted among the grasses. The Star Grass (*Hypoxis hirsuta*), which is not a grass, may do well here. Others recommended are the Indian Strawberry (*Duchesnea indica*), with strawberry-like foliage, the Blue-eyed Grass (*Sisyrinchium*

*angustifolium*), the Shooting Star (*Dodecatheon Meadia*), Bluets (*Houstonia caerulea*), English Daisy (*Bellis perennis*), and the small Red-seeded Dandelion (*Taraxacum erythrospermum*).

*Plains Terrarium:* This needs full sun, a light sandy well-drained soil, neutral or slightly limy, and not much watering, although care should be taken not to let the soil become too dry. Many interesting and easily grown plants belong here. Two ferns, the Purple Cliff Brake (*Pellaea atropurpurea*), and *Woodsia obtusa*, are good. A fern ally, popularly known as Resurrection Plant (*Selaginella lepidophylla*), with distribution from Texas southward, should live for some time. A few grasses are suitable, such as species of Brome Grass (*Bromus*), and the Triple-awned Grass (*Aristida purpureum*). Other grasses naturally growing in dry soil may be tried. A few cacti are desirable, though these are more conspicuous in the desert. Among the plains species of Cacti are two Mamillarias (*Mamillaria vivipara* and *M. missouriensis*), both small, nearly globose plants, *Homalocephala texensis*, a turnip-shaped form, and *Echinocereus reichenbachi*, a cylindrical species. Small specimens of the Eastern Prickly Pear (*Opuntia vulgaris*), and the Prickly Pear of the Middle West (*Opuntia Rafinesquii*), may be included. A great many bulbous plants belong here, and some will do well indoors. The Spring Star Flower (*Brodiaea uniflora*) from Argentina, with onion-like foliage and odor and star-shaped bluish-white flowers, can be obtained from some dealers, and is one of the "Cast-iron plants" in its ability to withstand hard usage. Bulbs should increase in number from year to year. Another very good plant if it can be obtained is the Cape Cowslip (*Lachenalia tricolor*), from the Cape of Good Hope, with mottled leaves and drooping yellowish flowers which resemble to a striking degree the flowers of the English Cowslip (*Primula veris*). Like *Brodiaea* the bulbs flower every year and increase in number. The Sand Lily (*Leucocrinum montanum*), very characteristic of the plains region, is also good. Other bulbous plants well worth trying, all natives of the plains of the western U. S., are two species of Onion (*Allium Helleri* and *A. mutabile*), *Androstephium caeruleum*, with light blue flowers of daffodil form, *Cooperia Drummondii*, with white lily-like flowers, *Nemastylis acuta*, a member of the Iris Family with sky-blue flowers, and *Nothoscordum bivalve*, with white, yellow-throated, lily-like flowers.

*Desert Terrarium:* Needed are a sandy, well-drained, calcareous soil, spare watering and full sunlight. It should not be nec-

essary to cover this terrarium. Here belong a great many evergreen plants of fleshy texture and often grotesque form. Suitable plants may be actually native to deserts or they may not, but in either case their habit of growth entitles them to be here included. Especially characteristic are the cacti in many forms. Those mentioned in the plains terrarium can be grown here, and there are many others. The Leafy Cactus (*Pereskia grandifolia*) is very easily grown and is very interesting. It is a large, slender branched form, but small cuttings can be used. Unlike all other cacti the genus *Pereskia* has well developed leaves, but these are yellowish-green, as in the mistletoe, in contrast to the bright green of the stems. The stems evidently perform most of the work of photosynthesis. The plant seems to be a transition form between plants with bright green fully functioning leaves and those in which the leaves are wanting or reduced to small non-green scales. The Christmas Cactuses (*Zygocactus* and *Epiphyllum*) are well known house plants with much flattened stems. The Orpine Family (*Crassulaceae*) contains a large number of plants of desert aspect, especially in the genus *Sedum*. The Common Stonecrop (*Sedum acre*) is a small trailing plant of easiest culture. Other genera of this family deserving notice are *Crassula*, *Sempervivum*, especially the Cobweb Houseleek (*Sempervivum arachnoideum*) and the Common Houseleek (*Sempervivum tectorum*), and *Echeveria*, the Common Hen-and-Chickens (*Echeveria secunda*) being especially good. Both *Sempervivum* and *Echeveria* produce offsets in large quantity. In the first these offsets are attached to long stolons, whereas in the latter they are sessile, remaining under the parent plant like chicks clustered under a hen. The Carrion Flower (*Stapelia variegata*), a cactus-like plant of Africa, can be obtained. It belongs to the Milkweed Family. The Succulent Groundsel (*Senecio succulentus*), a composite of South Africa, with fleshy cylindrical joints, is sometimes grown, and is easy. Aside from the succulents there are species with woolly stems and leaves characteristic of dry regions. Of these two may be mentioned. The Edelweiss (*Leontopodium alpinum*), famous flower of the Alps, a composite, though not found in deserts is exactly suited to desert conditions, and is now offered by dealers. Another one of easy cultivation is the Wild Silver Sage (*Artemisia frigida*) of the western U. S. This is a silvery plant with odor of sage, very common in cultivation. All these plants of the desert terrarium may be studied as typical xerophytes.

## AN OBJECTIVE ESSAY EXAMINATION IN CHEMISTRY\*

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The so-called essay type examination has had a wide and general use in testing the results of teaching. There are, however, two outstanding limitations to this type of examination. In the first place, only a limited sampling of the material to be tested can be covered in the time usually allowed; and in the second place the scoring of student responses usually lacks objectivity. To overcome these two limitations, the so-called new-type objective-tests have been developed, which do not have these limitations. However, the new-type objective-tests have in turn, certain limitations without the advantages of the essay-type test. For example, the construction of the new-type objective-test requires much time and skill. Unless great care and skill are used in constructing it, there is a serious danger of making the items too factual, so that the ability to do well upon the examination depends too largely upon memorisation of isolated facts, rather than upon a reasoned understanding of their meaning or relationships. The essay-type examination can be constructed in less time and with less skill.

A study of the literature that deals with test construction gives the impression that the essay-type examination can not be scored objectively. In fact, questions are often classified as either objective- or essay-type. If by some method the essay-type examination can be scored objectively, many of the advantages of both types of test will be combined in one test. The purpose of this article is to show that the so-called essay-type question can be constructed and a scoring procedure for evaluating student responses to those questions can be used which renders the essay-type examination highly objective. This is accomplished by designing each question so as to test a definite type of student behavior, and then scoring the response to the question by using that specific type of student behavior as the standard of evaluation. Incidentally the essay-type examination to be described also affords a more adequate sampling of content. The

\* In the June 1933 issue of this journal, pages 590-595, under the heading TESTS IN BIOLOGY, R. W. Tyler discusses the uses and the nature of well-constructed examinations. The reader who is interested in the general problem of testing will find it to his advantage to read Dr. Tyler's article which logically precedes the present discussion.

construction and scoring of the objective essay-type examination, which is about to be described, involves the following steps: First, for the purpose of testing, a particular teaching objective is selected. Second, this objective is defined in terms of the desired student behavior. Third, situations are collected in which students will have a good opportunity to reveal the presence or absence of the selected objective. And fourth, after the presentation of these situations to students, their responses to the situations are evaluated with the desired student behavior used as the standard.

One important objective in teaching chemistry is teaching students to use the principles of chemistry in interpreting chemical phenomena. There are other important objectives in teaching chemistry, but the test to be described is designed to test students' understanding of selected principles of this subject. It has been assumed that a student understands a chemical principle if he solves a problem (question for solution) when the accepted solution demands the use of the chemical principle. Recalling a memorized solution to a problem does not prove that a student understands the principles which must be used in obtaining that solution. Mere recall of the statement of a principle does not prove that the principle is understood. Neither is the student required to make predictions in order to demonstrate his understanding of a chemical principle. However, if a chemical situation is presented, the student can demonstrate his ability to use a specific principle by explaining, in terms of the principle and according to the demands of the question, what has taken place in the situation presented or how or why it has so occurred. In brief, therefore, it is assumed that a student understands a principle of chemistry if and when he can use it in explaining chemical phenomena in which it is involved.

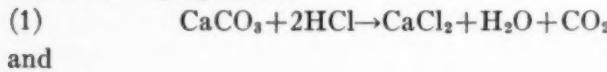
To develop a practicable method of testing, it is important to use testing situations which (1) are easily set up and which (2) are readily comprehended by students. A solution to this problem eventually developed when it was noted that many principles of chemistry must be used in answering the questions suggested by a chemical situation as expressed by a single chemical equation. A thrill comes with the realization that so much chemistry is unlocked by following the suggestions found in a single equation. For example take the following equation:  $\text{FeCl}_2 + \text{Zn} \rightarrow \text{ZnCl}_2 + \text{Fe}$ . What principles of chemistry that would be suitable for testing are suggested by this reaction it-

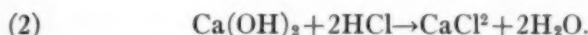
self and by the substances which are used in the reaction? Among the suggested principles are the following: (1) Displacement, or the Electro-Chemical Series (2) Theory of Ionization (3) Change of State,—Zinc can be changed at ordinary pressures from solid to liquid to vapor within a reasonable range of temperature—(4) Energy changes accompanying change of state (5) Oxidation and Reduction. The following questions are some that might be asked in order to determine whether or not a student understands oxidation and reduction:

1. Rewrite the above equation in ionic form indicating the involved changes in electrons.
2. Which element or elements has or have been oxidized?
3. Select an element which has been oxidized and explain the changes in its atomic structure in terms of electrons.
4. What element or elements has or have been reduced in the above equation?
5. Select an element which has been reduced and explain the changes in its atomic structure in terms of electrons.
6. What elements, if any, have not undergone oxidation and have not undergone reduction?
7. Cite an example of a reducing agent from the above equation.
8. From your study of the given equation, would you say that the amount of oxidation is more, less or the same as the amount of reduction. Prove your answer.

Two reactions involving practically the same elements have several advantages over one reaction in test construction. Certain problems for testing certain principles are suggested by each reaction and other problems are suggested by comparisons between the two reactions. The range and variety of testing situations is thus greatly increased without adding greatly to the information necessary to be given to the student. To accompany the reaction given above, the following is suggested:  $2\text{FeCl}_2 + 3\text{Cl}_2 \rightarrow 2\text{FeCl}_3 + 27,900 \text{ calories}$ . This equation at once suggests the following principles: (5) Oxidation and Reduction (6) Constant heat of Formation (7) Energy Change (8) Law of the Influence of Heating (9) Kindling Temperature etc. Taking these two equations together gives a basis for testing the nine chemical principles listed. The first twelve principles to be listed presently could also be tested by these equations.

Based upon this plan a test was constructed and tried out with several high-school and college classes. The test used the two following equations:





In the manner indicated, these equations served as the basis for testing student understanding of the following twenty selected principles of chemistry:

1. Properties
2. Atoms and the Theory of Atoms
3. Formulae, their Interpretation and Use
4. Molecules and the Theory of Molecules
5. Law of Component Substances
6. Law of Constant Composition
7. Law of Conservation of Matter
8. Interpretation and Use of Chemical Equations
9. Avogadro's Theory
10. Kinetic Theory
11. Atomic Structure (Electron-Proton Theory)
12. Chemical Affinity and Valence
13. Classification of Inorganic Substances
14. Theory of Ionization or of Electrolytes
15. Neutralization
16. Boyle's Law
17. Charles' Law
18. Reversibility and Mass Action
19. Decomposition
20. Metathesis

All needed information, such as atomic weights, Avogadro's number, and definition of terms were given to the student so that he was tested *only* on his ability to use the principles of chemistry involved and *not* on the amount of material which he had memorized. The following sample illustrates the type of questions:

- Number 3. How many atoms make up a molecule of calcium carbonate ( $\text{CaCO}_3$ )?
- Number 5. What is the molecular weight of:  
a. Calcium carbonate ( $\text{CaCO}_3$ )?  
b. Calcium chloride ( $\text{CaCl}_2$ )?  
c. Calcium hydroxide ( $\text{Ca}(\text{OH})_2$ )?  
d. Hydrogen chloride ( $\text{HCl}$ )?  
e. Water ( $\text{H}_2\text{O}$ )?  
f. Carbon dioxide ( $\text{CO}_2$ )?
- Number 12.  
a. From equation (1) it can be seen that one gram molecular weight of carbon dioxide is formed. This is how many grams of carbon dioxide?  
b. How many molecules make up this weight? (Read this question carefully).  
c. How many gram molecular weights of hydrogen chloride are represented by equation (1)?  
d. How many tiny molecules of hydrogen chloride does it take to make up the weight of hydrogen chloride represented in equation (1)?

Number 13. Using the fundamental ideas in the Atomic, Molecular and Kinetic Theories of Matter, explain briefly how, why and what has taken place in the reaction represented by equation (2)

Since each question tests the understanding of certain principles of chemistry, student responses were evaluated by using the tested principles as a standard. Each grader was asked to judge from each answer how well the student understood the specified principles involved. For example, questions 3 and 5 were used to help determine the students' understanding of principles 2, 3, and 4 as listed above; question 12 tests principles 3 and 9, and question 13 tests principles 2, 4, and 10. Each principle tested in each question was scored on the basis of five. From the answer given by the student the grader judged whether the student should receive 0, 1, 2, 3, 4, or 5 on his understanding of each principle tested by each question. This may be further illustrated by question 13 above. Two student replies to this question and the average of the scores which they received on each principle as judged individually by six graders follow:

Answer of student 33 to question 13.  $\text{Ca}(\text{OH})_2$  has entered into the reaction with HCl. The calcium combined with two atoms of Cl to form  $\text{CaCl}_2$ . The H of HCl combined with the  $(\text{OH})_2$  group to form water. Heat and energy have been liberated.

The scores received by this answer on principles 2, 4, and 10 were respectively 3, 2, and 1.

Answer of student 45 to question 13. One gram mol. wt. of  $\text{Ca}(\text{OH})_2$  has reacted with two gram mol. wts. of HCl to form 1 gram mol. wt. of  $\text{CaCl}_2$  and two gram mol. wts. of  $\text{H}_2\text{O}$ .

The tiny atoms making up the molecules according to the atomic theory are responsible for making the reaction go because they enter into the reaction. Molecules are moving about at a rapid speed bumping into each other (molecular theory).

The scores received by this answer on principles 2, 4, and 10 were respectively 5, 4, and 4. Again it may be pointed out that each grader in scoring each student response for a given principle in a given question decided how well that student understood the principle. Although the principle was the standard of evaluation, the definition of each principle was left to each grader.

It so happens in this examination that a student receiving full credit of five points on each principle tested by each question would receive a total of 260 points, the maximum score possible.

One method of arriving at a per cent score for each student is to divide the points received by 260, and this per cent value is called the *score on points*.

For comparing a student's understanding of one principle with his understanding of another, the number of points received on each principle can not be used, since the total points possible for one principle was not necessarily the same as the total points possible for another principle. That is, a student was given more opportunity to demonstrate his understanding of some principles than of others. Comparison between scores was made possible by converting the score in points for each principle to a score in per cent. That is, the number of points received for each principle is divided by the total points possible for that principle. The average of the per cent scores received on each principle serves as the *score on principles* for the student.

To be objective, a test must be capable of being scored with practically the same results by a number of qualified individuals. In order to determine the objectivity of the test described, the following plan was used: Ten papers, chosen by chance, were used as a sample of student response to the test. The scoring of each paper required a maximum of 52 subjective judgments on the part of the grader. The ten papers, therefore, required a maximum of 520 judgments by each grader. These ten papers were scored independently by each of the six qualified graders. The correlations between the average of the scores given by the six graders and the scores given by each grader were determined. These six graders may be characterized in the following manner:

*Graders F and T* are test technicians of the Division of Accomplishment-Tests of Ohio State University and have taught science on the secondary level. The construction of science tests on both the secondary and on the college level is part of their present work.

*Grader D<sub>1</sub>* is a research fellow in chemistry. His teaching experience has been limited to assisting in college chemistry.

*Grader D<sub>2</sub>* is now completing research for the degree of Doctor of Philosophy in chemistry. He has taught secondary science for two years. This experience was followed by graduate work for a Master's degree in chemistry and with seven years work in industrial chemistry.

*Grader D<sub>3</sub>* has for several years been a professor of general college chemistry.

*Grader H* has had no teaching experience except as a student

TABLE I  
THE TEN STUDENTS' UNDERSTANDING OF CHEMICAL PRINCIPLES AS ESTIMATED BY SIX GRADERS

Grader	Chemical Principle Numbers One to Twenty										Correlation with mean	Grader								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
H	63	68	74	63	73	63	76	70	60	19	57	47	58	34	33	55	44	50	55	51
F	60	71	72	62	74	59	63	76	60	26	55	53	52	30	20	68	54	26	38	.995
D <sub>1</sub>	65	74	82	64	80	71	72	73	59	30	61	62	62	32	44	70	42	38	56	44
T	52	74	77	62	86	75	70	69	62	25	59	66	50	32	24	63	34	28	44	.860
D <sub>2</sub>	54	66	67	58	78	61	69	76	68	34	52	64	44	26	20	59	46	53	56	.885
D <sub>3</sub>	64	69	79	52	81	72	71	78	63	13	58	68	56	30	10	80	64	36	56	.963
Mean	60	70	75	60	79	67	70	74	62	24	57	60	53	31	25	73	47	38	51	.44
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

The above scores have been rounded off to the nearest whole number. The mean score for each principle and the correlations were computed before the above numbers were rounded off.

teacher. He has done graduate work in chemistry and in education. The letters used above will be used in indicating results obtained from the scoring of the ten papers by each grader.

Table I summarized the ten selected students' understanding of chemical principles as judged by each of the six graders. Table II summarizes the *scores on points* given each of the ten students by each grader, and Table III lists the *scores on principles* given each student by each grader. These tables also show

TABLE II  
THE TEN STUDENTS' SCORES ON POINTS AS ESTIMATED BY EACH OF THE SIX GRADERS

Grader	Student Numbers										Correla-tion
	3	9	15	21	27	33	38	45	51	58	
H	43.8	55.0	64.4	34.2	60.9	69.6	84.2	70.3	27.5	93.9	.988 H
F	69.3	53.0	68.4	35.7	58.7	64.6	79.3	70.7	24.6	93.0	.937 F
D <sub>1</sub>	48.0	58.2	71.6	35.4	64.6	68.1	84.6	79.7	38.8	96.8	.981 D <sub>1</sub>
T	38.1	51.2	73.1	38.4	63.8	64.8	85.4	72.3	31.2	96.9	.967 T
D <sub>2</sub>	42.3	56.0	64.3	34.6	62.0	66.2	82.0	67.3	29.2	97.3	.993 D <sub>2</sub>
D <sub>3</sub>	50.3	55.7	71.4	38.8	60.8	66.7	80.2	70.0	33.4	96.5	.987 D <sub>3</sub>
Mean	48.6	54.9	68.9	36.2	61.8	66.7	82.6	71.7	30.8	95.7	Mean
	3	9	15	21	27	33	38	45	51	58	

The correlation column refers to the correlation of each row with the mean.

TABLE III  
THE TEN STUDENTS' SCORES ON PRINCIPLES AS ESTIMATED BY EACH OF THE SIX GRADERS

Grader	Student Numbers										Correla-tion
	3	9	15	21	27	33	38	45	51	58	
H	41.3	38.0	56.9	20.2	45.7	64.6	83.8	80.2	21.7	95.1	.967 H
F	40.8	41.6	60.6	20.6	43.7	42.3	73.5	72.4	17.0	94.3	.976 F
D <sub>1</sub>	46.8	50.0	64.3	21.0	53.3	53.2	88.0	87.1	29.4	97.0	.989 D <sub>1</sub>
T	33.0	40.5	62.4	24.1	52.4	47.3	81.2	76.8	26.6	96.1	.989 T
D <sub>2</sub>	35.0	44.4	59.2	19.7	70.7	49.1	81.0	72.7	25.0	96.2	.951 D <sub>2</sub>
D <sub>3</sub>	50.7	45.2	69.1	22.4	53.3	58.0	81.1	76.8	28.6	94.0	.971 D <sub>3</sub>
Mean	41.3	42.6	62.1	21.3	53.2	52.4	81.3	77.7	24.7	95.4	Mean
	3	9	15	21	27	33	38	45	51	58	

The correlation column refers to the correlation of each row with the mean.

the correlation between the results obtained from the scoring of the ten papers by the various graders with the mean results of the six graders.

These correlations\* are very high; which indicates that the scoring of the described test, which used no form of suggested answers, was highly objective and that the estimate of any one grader is practically as good as the mean estimate of six graders. This means that the so-called essay-type examination can be constructed so as to be scored objectively by consciously selecting test situations to cover specific objectives and by evaluating the answers with reference to these objectives.

\* The correlation coefficients were computed by the Pearson Product-Moment Coefficient of Correlation Formula.

## A TENTATIVE SCHEME FOR TEACHING POLITICAL GEOGRAPHY IN THE HIGH SCHOOL\*

BY KATHARINE CALLOWAY

*Kelly High School, Chicago*

The purpose of this paper is not to discuss the educational value of political geography for secondary school pupils, or the comparative claims of political, physical, and economic geography for place in the high school curriculum, much as these matters deserve consideration, but to present a tentative scheme for the teaching of political geography in the high school which may be of interest to others engaged in experimentation in this field. The proposed course is designed for use with pupils who have completed one year of economic or commercial geography. It was suggested by, and is based upon an earlier course, entitled "Political Geography of the Major Nations," originating with a committee of the National Council of Geography Teachers, but includes such changes and modifications of the earlier course as seem advisable in suiting it to the needs and background of the experimental class.

The course deals chiefly with the political problems of eight major nations, the United States, Britain, France, Germany, Japan, Italy, Russia, and China. It comprises an introductory unit, eight units treating aspects of the political geography of the foregoing nations, and a summarizing unit.

The purpose of the introduction is three fold, (1) to initiate the class in specific methods of work, (2) to give an understanding of the meaning of the terms "Major Nations" and "Political Problems," and (3) to present a type political problem. By means of exercises that furnish tables of area, population, value of foreign trade, and extent of colonial possessions of various independent countries, pupils compute the number of representatives that the eight nations, respectively, might send, on each of these bases, to an international conference. These exercises make clear that, although a nation's rank as a "major nation" is determined by no exact set of ratings, in general (1) its resources in people, their number and character, (2) the natural resources that help to furnish its economic strength, (3)

\* Abstract of a report presented to the Geography Section of the Central Association of Science and Mathematics Teachers, December 1, 1933. A part of this report was published in the *Yearbook* of the National Society for the Study of Education, February 1933.

the value of its foreign trade, and (4) the role that a nation plays in world affairs must all be considered in rating a country as "major." The type problem presented in the introductory unit is one that involves cooperation among nations, "The Development of the North Atlantic Ice Patrol." From assigned readings the class prepares to discuss the following topics: (1) The iceberg danger to North Atlantic shipping, and the reason why it concerned several nations; (2) the political agreement reached by these nations concerning a means of lessening this danger; (3) the problems the government of the United States faced in carrying out its part of the agreement, and the ways in which it has solved them. The study of this problem acquaints pupils with three ideas, namely, that political problems are concerned with *governmental* activities; that political problems are outgrowths, to some degree, of natural conditions; that political geography is a study, in part, of the ways in which nations take natural conditions into account in trying to solve political problems.

In the subsequent nine units of the tentative course selection of topics for class investigation has been made with these considerations in mind:

1. The politico-geographic quality of the topic, its concern with relationships between governmental activities or national pattern, on the one hand, and natural environmental conditions, on the other.

2. Balance in types of problems, the inclusion of problems dealing with internal as well as with external governmental affairs, and with methods of cooperation among nations as well as with situations of rivalry and dissension.

3. Comprehensibility and practicability, the use of problems that can be solved effectively by high school pupils by a treatment that encourages open-mindedness on their part rather than the immaturely opinionated mind.

The outline of the tentative course in its larger divisions follows.

#### POLITICAL GEOGRAPHY OF THE MAJOR NATIONS

- I. Introductory unit. The meaning of "Major Nations" and "Political Problems."
- II. Some aspects of the political geography of the United States. *A Motivating Exercise.* How many of the major countries are in the western hemisphere? What surprising fact about the United States does this discovery call to your attention? One problem to be considered, therefore, is "*How have people of the United States utilized*

*natural environment in attaining a political rank higher than that of other American countries?"* In considering this problem, the topics that follow will be found helpful.

*Topics for investigation by class:*

1. The development of a democratic type of government in the United States as related to occupancy of extensive, humid, fertile plains.
2. United States federal income as related to utilization of the nation's natural resources.
3. The geography of government regulation in the United States of industry and commerce.
4. The geography of the Consular Service, as an agent of international intercourse.
5. The Expansion Policy of the United States in relation to economic conditions here and in the dependencies.
6. Geographical bases for differences in the political status of the United States and of other American republics.
  - a. The political significance of the presence in a nation of natural barriers to communication, as in Central America.
  - b. The political significance of the possession by a nation of large areas of uncultivable territory, as in Mexico and Brazil.

**III. Some aspects of the political geography of Britain**

1. The political pattern of Britain with relation to contrasting areas for trade and contrasting natural conditions in those areas.
2. Geographical bases for the high rank of Britain as a colonizing power.
  - a. An analysis of homeland conditions: industrial development and population crowding.
  - b. An analysis of the British colonial pattern.
3. Political regulations of commercial relations among the Commonwealths of the British Empire.

**IV. Some aspects of the political geography of France**

1. National unity in relation to economic and natural conditions of the various divisions of the country.
2. International considerations regarding conditions on French frontiers: The economic relations of the "International Triangle"; international interest in Alpine power resources.
3. National defense policy in relation to the character of national boundaries.
4. Some geographical considerations involved in the political unity and the administration of the French colonial system.
  - a. The question of suitability of colonial territory for expansion of French population.
  - b. The relation of colonial trade to French industries.
  - c. Conflicts with other powers arising from colonial expansion.

**V. Some aspects of the political geography of Germany**

1. Political pattern in relation to natural environmental conditions of Germany.
  - a. The geographical basis for the union of the German States, comprising contrasting areas for trade.
  - b. An evaluation of present German boundaries with regard to their crossing of unit-economic areas, and their natural characteristics for defense.

**VI. Some aspects of the political geography of Italy**

1. The political status of Italy in relation to use of power resources.
  - a. The status of the Italian States before the age of steam.
  - b. The status of Italy during the age of steam.
  - c. The status of Italy since the rise of the use of hydro-electric power.
2. Geographical bases of the colonial ambitions of the Fascist State in the Mediterranean area.
  - a. Italian population density in relation to natural resources in Italy.
  - b. The economic character of Italian colonies as compared with those of Britain and France.
  - c. Central position of Italian peninsula with reference to Mediterranean trade.

**VII. Some aspects of the political geography of Japan**

1. A comparison of the political status of Japan with that of Italy in relation to the use of power resources.
2. A comparison of the geographical bases for the expansion policies of Japan and Italy.

**VIII. Some aspects of the political geography of China**

1. Geographical bases for the age-long survival of the Chinese State.
2. The problem of establishing political unity in China as related to geographical factors there. (Build upon understandings gained in Topic II, 6.)

**IX. Some aspects of the political geography of Russia**

1. Geographical considerations in the expansion policy of Pre-war Russia.
2. Development of the loosely-federated Soviet Union in relation to occupancy of vast and physically diverse territory.
3. Soviet control of industry and commerce in relation to
  - a. Utilization of Russia's natural resources.
  - b. Foreign Trade.

**X. Summarizing Unit:**

The geography of some agreements, treaties, and conferences among nations, which have grown out of the political situations and their geographical bases, discussed in the study of the foregoing units.

The suggested treatment of one topic, namely, "Consular Service as an Agent in International Intercourse" follows:

1. Motivation by means of map work.
  - a. The making of a dot map, as a class enterprise, to show the distribution of United States consuls throughout the World. (Congressional Directory)
  - b. The making of a dot map, using dots of various colors, to show the distribution of foreign consuls in the United States.
  - c. The consideration of questions growing out of map findings, such as:
    - (1) Why does the United States send many consuls to Canada, Mexico, Western Europe, and Eastern Asia?
    - (2) Why do relatively few United States consuls serve in countries of the southern hemisphere?
    - (3) Why are the greater number of foreign consuls located in eastern United States?
2. Exercises to aid investigation.

- a. A study of the duties of consuls.
- b. An analysis of the distribution of United States consuls.
  - (1) With relation to distribution of United States foreign trade and to the natural conditions basic to such trade.
  - (2) With relation to movements of population between countries.
  - (3) With relation to geographical types of cities.
- c. An analysis of the distribution of foreign consuls in the United States.
  - (1) With relation to areas of the United States which give rise to much trade and to the natural conditions underlying such trade.
  - (2) With relation to distribution of foreign population in the United States and to economic conditions underlying such distribution.
  - (3) With relation to given types of cities in the United States.
3. Conclusions.
  - a. Formulating answers to the questions arising from the map work.
  - b. Summarizing conclusions in regard to geographical bases for consular service and its importance as an agent in international intercourse.

From the use of this course in political geography with secondary school pupils, the teacher may expect outcomes such as the development of (1) specific abilities to identify and to understand multi-step relationships existing between governmental practices and problems, economic conditions, and aspects of the natural environment; (2) the ability to use understandings of such relationships, among other criteria, in forming personal opinions upon certain political issues; (3) an open-minded attitude toward the political biases of nations with contrasting environmental backgrounds. Further investigation, with modification of this scheme, will doubtless be necessary to insure these outcomes.

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#### A SUGARING-OFF AT HUDSON

The spring meeting of science and mathematics teachers held at Hudson, Ohio, March 17 was a great success.

"A wonderful day in a wonderful setting," writes one of those who attended. "The conditions were perfect—sun warm, a south wind, the sap running fine, buds not out too far, hard to find *Erigenia Bulbosa*. Dr. Vinal at his best. A large attendance—over one hundred at the morning meeting and noonday luncheon, over 300 reservations for the evening sugar-off. The attendance and interest greater this year than last."

Every community should have its local organization of teachers of the basic sciences. In such meetings leaders are trained, new movements started, cooperation and coordination perfected. Plan and hold such a meeting then report it for *SCHOOL SCIENCE AND MATHEMATICS*.

## CAMPING, AN IMPORTANT PART OF A CHILD'S EDUCATION IN ELEMENTARY SCIENCE\*

BY WILLIAM GOULD VINAL

*Western Reserve University, School of Education*

Camping was the original American school. Most of our American ideals and virtues date back to our close contact with nature. The rifle, the axe, and the plough were the chief tools of learning. Then came the period of congregating in large centers and the values of camping were lost.

Perhaps it was just before the Gay Nineties that the call of the wild again made itself felt. As early as 1861 Mr. Frederick William Gunn, the founder of the Gunnery School in Connecticut, started a summer camp for boys. It was not until 1881 that the movement got underway. Camp Arey was originally established in 1890 as a Natural Science Camp for boys and in 1892 was the first camp to accept girls. By 1910 a legion of private and organization camps had demonstrated the advantages of the outdoor camp life.

Camping as an adjunct to the public school system began in 1912. The Visiting Nurses' Association of Dubuque, Iowa in co-operation with the Board of Education, established a summer camp for malnourished school children. In 1919 through the efforts of Major Frank L. Beals normal children were admitted to Camp Roosevelt an integral part of the Chicago Public School system. Today it is not unusual to find camps for public school children maintained by Boards of Education.

The first school to train nature counselors in camp was established in 1920, the same year that Nature Guiding started in our National Parks. In April, 1921, the Nature Study Review printed its first camp number. March 14, 1924 the Camp Directors Association was founded. All of these were essentially organized efforts with camp education as the central theme.

If we were to classify camps today we would find nutrition camps, camps for tuberculous children, camps for underweights, camps for diabetics, camps for cripple children, private camps for the rich, and camps for normal children from homes of average incomes maintained by the scouting organizations, the Y. M. C. A. and Y. W. C. A. If a child is sufficiently malnourished or if he is rich enough he can go to camp. The organiz-

\* Illustrated and read before the Elementary Science Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

ing of camps for the great army of public school children is in its beginning stages.

Mark this! The day is coming, if it is not already here, when the great American public is going to stand back of "camps for every child" in the same way that they consider our public school system as the bulwark of America.

And at this time we can not be unmindful of the 1468 camps or the Civilian Conservation Corporation the biggest peace time army in the nation's history. Last summer, Cleveland, and I presume that it is true of other cities, found it cheaper and better economy to transfer some of its boys and girls of the bread line to a city camp hastily constructed by men on its city work farm. Next year, with community chests staggering many more municipalities are going to come face to face with the issue. The camp movement is bringing boys and girls into the very center and fountain head of nature education.

All of this is but a brief history of the camp movement. Every teacher is certainly conscious of the present day situation. The camp movement is essentially a back to nature and a back to health movement. When these youngsters are suddenly thrust into camp do they make use of the science education which you teachers have given them or do they do things in spite of what you have taught? What have you taught that might prepare them for health in camp? What have they learned in the way of leisure time activity in the woods? What attitudes and skills do they have in the way of citizenship in the open? Have you provided them with the kind of character that will work in camp? This is a good time to take account of stock in our science teaching and tune it up to present day needs.

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#### NEW VARIETY OF WHEAT NOT LIKED BY GRASSHOPPERS

Grasshoppers, which have been making unusual pests of themselves in the Wheat Belt during the past few summers, are expected to be "highly insulted" when they hit fields of the new wheat variety known as "Ceres." Dr. L. R. Waldrön, plant breeder of the North Dakota agricultural experiment station, has received letters from many farmers, agreeing that for some reason as yet unknown Ceres wheat is not liked by the' hoppers. Due to this 'hopper-resistant quality, Ceres is said to have outyielded other Dakota bread wheats by as much as thirty per cent during the recent grasshopper years.

Ceres was originally bred with the special objective of getting a drought-resistant, rust-resistant, high-yield wheat. Its unpalatability to 'hoppers appears to have been uncalculated—but welcome, nevertheless.—*Science Service*

## CALCULATE BY EIGHTS, NOT BY TENS

BY E. M. TINGLEY

*Oak Park, Illinois*

*Our minds and the nature of things:* Because of our two eyes and two hands we prefer to halve and double things. Therefore we should also use measures and arithmetic containing only even factors. Eight gives the best even scale or base for this arithmetic.

Our minds prefer the simply easy even operations on things, repeated many times where necessary. We divide things evenly into halves with our eyes. We judge equality of weights with our two hands. We split differences to close trades. Surely these are facts, not opinions.

We like to halve and double things. But we are now forced by our decimal arithmetic to calculate about these even halves and doubles with fifths and odd and prime fives.

Our minds and the nature of things will never change in respect to halves and doubles. We cannot change our minds to fit our present decimal arithmetic. We must adapt our arithmetic to our minds. Therefore we must correct our arithmetic and measures from the scale ten to the scale eight.

The old proverb about the fit of a square peg in a round hole must be revised to include the lack of fit of our five-sided arithmetic in our foursquare minds.

As pairs are favored in our minds and in all nature our arithmetic should favor pairs. By using the base eight, the product of pairs, our calculating system will best correspond with many features of nature.

*Psychologists and arithmetic:* Our most important mental tool after language is arithmetic. What is the best base for our arithmetic? Analysis of the relations of our minds to measures and the arithmetic base properly belongs with the psychologists, scientists of the mind and its activities regarding things and numbers. But where can a psychologist be found who is aware of this? We are now afflicted with the base ten but a greater injury is done in the neglect of psychologists to determine what number scale allows the mind to work most efficiently, that is, with least effort.

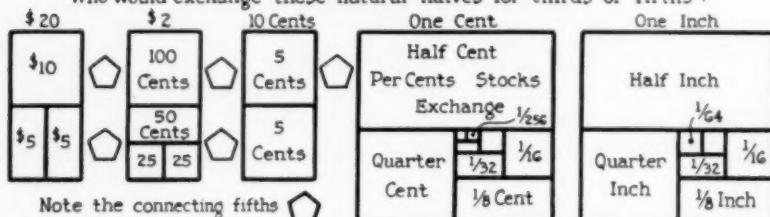
If a change of base is assumed impossible where before did an impossibility deter a real scientist from study of a subject

## LET'S CALCULATE BY EIGHTS, NOT BY TENS

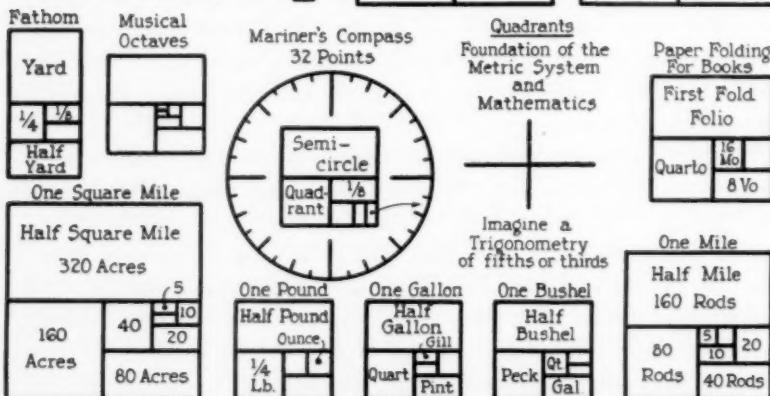
## PICTURES ARE BETTER THAN NUMBERS OR WORDS

To show that we divide things in halves - again and again - by choice - because we have two eyes and two hands.

Who would exchange these natural halves for thirds or fifths?



Note the connecting fifths



Extend and make exclusive this easy halving and doubling of measures to force arithmetic to count and calculate by eights in place of tens. End our mental waste between our natural even measuring scales of halves and the decimal counting and calculating scale containing the odd fives.

Who can make these troublesome thirds and fifths accurately without mechanical aid? Fifths, difficult for our hands and minds, are always present in our decimal and metric systems.

Psychologists should measure and declare our eternal preference for halves above thirds or fifths. but they neglect their great opportunity. Psychologists should condemn the metric system with its confusing fifths. Arithmeticians and educators wrongly favor the scale of twelve containing in the place of five the equally offensive odd and prime three.

E. M. Tingley, 221 North Cuyler Avenue, Oak Park, Illinois, U.S.A.  
January 37, 3616 First Revision

## CHART I

These pictures show graphically how we like to halve or double things and measures.

Our land measure is a familiar example. We divide a square mile of 640 acres down to five acres by halving. Miles are usually divided into halves and quarters. Inches are divided down to sixty-fourths. Unfortunately there is no easy ratio between miles and inches.

We divide twenty dollars by halves to five dollars. The next divisor is five halves to give us two dollars.

Who divides anything into fifths or thirds of it can be avoided? Where fifths appear in money the divisions have been already made for us.

Quadrants formed by the horizontal and vertical seem a property of nature. Why not change our arithmetic base to correspond with nature?

## LET'S CALCULATE BY EIGHTS, NOT BY TENS

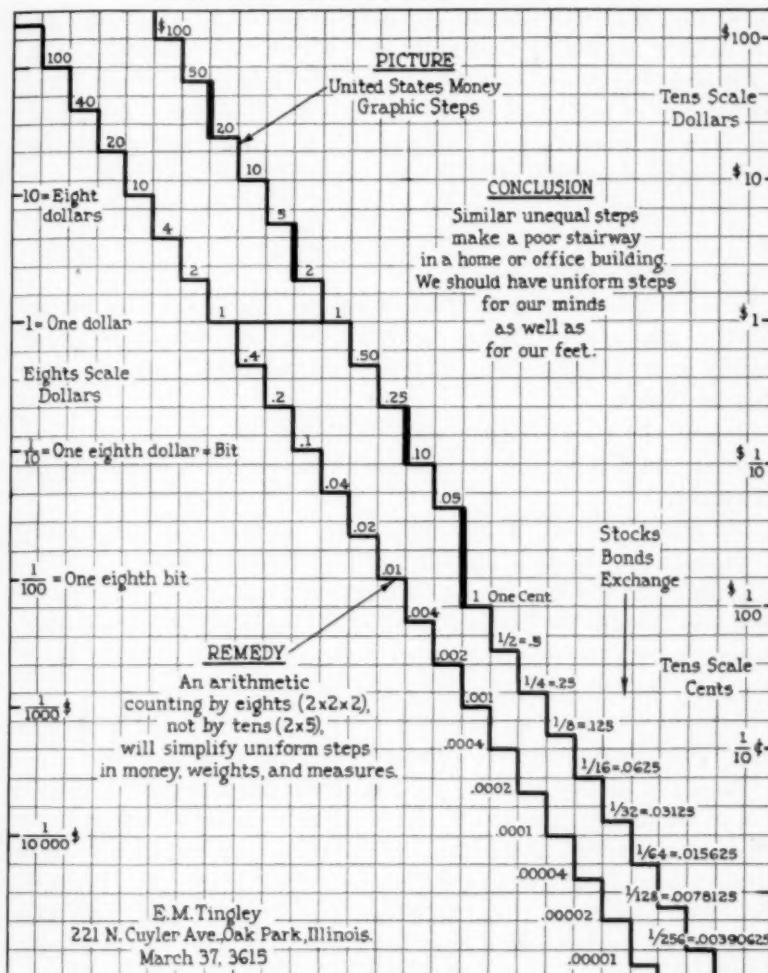


CHART II

This stair-step chart shows our preference for the natural easy even steps of halves. Our even U. S. money steps are interspersed with steps of fifths. This allows registration with the decimal scale to the right. Steps of fifths are avoided. Compare the number of U. S. money steps containing halves with those containing fifths. Easy halves are extended below one cent. They are accompanied with inconvenient decimal fractions.

Only easy steps of halves are shown on the left. They register completely with the eights scale to the left. Such steps and scale may be extended indefinitely.

The scales are logarithmic. A given divisor is measured downward the same distance in any part of either scale.

The ratio of the scale of eights to the scale of tens is .90309, the ratio of their logarithms.

and insistence in his conclusions. Psychologists, that vast influential and international group, students of the mind, should assert in their authority that eight is the best base for arithmetic.

*English weights and measures:* The principle merit of the English system of measures lies in the halving of many of its units, repeated in several cases. Otherwise it is a brain-tiring mixture of ratios having difficult relations with the decimal scale.

The preference to halve and double measures is shown graphically, pages 396-397, and this element that allows English measures to persist should be extended.

We now use a jumble mixture of halves, thirds, fourths, fifths, eighths, and tenths. With the base eight we would measure mainly in repeated halves, quarters, and eighths, and calculate only by eights.

Or put the case another way. With the change to the base eight accomplished who would use thirds or fifths in money or measures?

*The metric system:* This begins with quadrants of our earth, an inconsistent foundation for five and ten measures.

A time-serving expedient, without the sanction of psychologists, is the metric system, devised to closely connect metric measures with the decimal scale. Both are objectionable as they can never fit our preferred way of thinking.

Steps of five are too large for convenience and the odd and prime five discourages the repeated halving of these steps.

Our U. S. money is usually cited as the most perfect application of the metric principle. However analysis shows that here halves are favored and fifths are avoided. Our commonly-used money steps from twenty dollars to stock loan rates include seventeen steps. Of these fourteen are steps of two, two are steps of two and five, and only one is a step of five. The evidence here is plain that five should be discarded in favor of four or two twos.

Misguided legislation, without the benefit of the correct authority of the psychologists, is liable to further inflict on us the makeshift metric system and its eternal odd fives.

The metric system was made for the benefit of those who calculate, not for those who furnish the material for calculations.

*Mind efficiency in measures:* Modern civilization depends on labor-saving physical tools and labor-saving hand motions indicated by mechanical engineers.

No corresponding mind-saving improvements have been pro-

posed by mind engineers in mind tools, measures and arithmetic.

Efficiency engineers study and make simple the finger, hand, and body motions of a shop man. Their works and methods are recorded in many books. How much more important is it for mind engineers to make more simple and easy the mind operations in arithmetic and measures used everywhere and always.

For example, teachers find that it is necessary to use the idea of halves and quarters to introduce fractions to the child mind. Thirds are avoided as being more difficult than quarters. Teachers then complain bitterly that later these children must calculate about quarters with tenths.

With the base ten it is necessary to memorize 390 number facts. For the base eight only 250 number facts are required, a further saving in time and effort.

There is nothing divine or unchangeable in ten. To change arithmetic to the base eight we discard the figures 8 and 9 and write eight, 10. Page books and number houses to introduce the numbers to the public. These are sequence numbers involving little or no calculation. Substantially complete change to base eight numbers and measures should require about three generations.

*History of arithmetic:* In the early ages nature provided us with eight fingers for the best counting system, and two thumbs as pointers. Unfortunately our savage ancestors, being ignorant of the refinements in division, used their thumbs as counters also. The ten scale was started when simple finger counting was a great accomplishment. Counting came before multiplication or division.

Our savage ancestors started the scale of ten. Why should we continue to be afflicted by ten and its defenders?

Our present method of writing numbers was devised about 750 A.D. Eight centuries more were wasted before we found that decimal fractions could be written in the opposite direction from the integral numbers. We have been using decimal fractions only a little more than eight generations, a short interval in human history.

Shall we waste another eight generations before we use our intelligence to adopt the base eight to simplify a world language of measures, money, and numbers.

Since the future is greater than the past we should now make this change to eight for the benefit of future thousands of generations.

**TEACHING SCIENTIFIC METHOD****Article II: Problems for Developing Skill in  
Scientific Thinking**

BY ELLIOT R. DOWNING

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When we speak of the method of the scientist we usually have in mind that method by means of which noted scientists have discovered the laws of science—the inductive method of thinking. Confronted with a problematic situation it is recognized as such and the problem is clearly defined. Then the salient elements of the problem are selected, facts collected that bear upon them and these are studied to see if they will suggest any tentative theory to account for the phenomena observed. An hypothesis once formulated is tested by observation or by experiment, if possible, otherwise by inference to see if it will stand up under the test. If not another hypothesis must be sought and tested. On the basis of all the facts disclosed one reaches to a more or less correct judgment as to the causal relations that exist.

An excellent illustration of this type of procedure is seen in Kepler's (1571–1630) discovery of his laws. The motive for the attack on his problem was his desire to give help in a very practical matter. If one could foretell the exact position of the planets as seen against the background of the fixed stars, they would serve as the hands of a gigantic clock. The mariner, provided with a table of such positions, could tell what time, at the observatory, say at Prague, a given planet would be in a specific position. Then by noting the passage of some conspicuous star or of the sun across his meridian, he could tell his local time. He could then observe at what local time the planet would appear in the particular position. Thus knowing the difference in time between Prague and his location he would calculate his latitude or distance east or west of Prague.

The tables predicting such planetary positions were very inaccurate in Kepler's day. Tycho Brahe, the great Danish astronomer, had been disgusted in his early life to find that the time prophesied for a conjunction of two of the planets was wrong by a whole month. So he determined to devote his life to an accurate mapping of the positions of the fixed stars and an equally accurate determination of the position of the planets

among the stars, night after night, year after year. Kepler joined Brahe in this undertaking toward the close of Brahe's life and continued it after the latter's death. They mapped a thousand of the brighter stars and made hundreds of observations of the planets over a period of fifty years, measuring accurately their angular distances from several adjacent stars at each observation. Thus had been patiently accumulated a fund of exact facts.

But now before one could calculate where a given planet, like Mars, would be located among the fixed stars, a year or two in advance, at a particular time on a specific date, he must know around what point the planet was revolving, the shape and size of its orbit and the rate of the planets movement. Kepler was a convert to the Copernican theory that the planets revolve around the sun. The distance of Mars from the sun was known with fair accuracy as was also its time of revolution.

Up to Kepler's time astronomers had always assumed that the orbits of the planets were circles. He was forced to test this hypothesis by inference. He could not project himself out into space to observe the shape of the orbit, he could not experimentally change its shape to see what would happen. Knowing the position of the planet among the stars as Brahe had observed it on a particular date several years previously he could calculate where it would be a year or so later if its orbit were a circle. Then he had only to consult the record of observations to see if it had been in that position. He made such calculations repeatedly but the calculated position never agreed with the observed position. He was forced to conclude, therefore, that the assumption that the orbit of Mars is a circle, was contrary to fact.

Then he thought the orbits might possibly be ovals. Again he made his calculations for such orbits of various relative lengths and breadths. But still the calculated positions did not agree with the observed positions even approximately. So he guessed again. Suppose the orbit of the planet is an ellipse with the sun located at one of the foci. Again he goes at his calculations and now he is delighted to find that the calculated positions are coming much nearer the observed positions. He thought he was on the right track but still the agreement was not perfect.

So he made another bold assumption, namely, that the planet moved in its orbit more rapidly when nearer to the sun, less

rapidly when farther from the sun. Impressed with the idea of a uniformity in nature, he further assumed that the line from the sun to the planet covered equal areas of the orbit in equal times. He again spent months on the new calculations and finds at last that calculated positions and observed positions do agree reasonably well. So he announced his first two laws: (1) The planets revolve about the sun in elliptical orbits, the sun at one focus; (2) that the radius covers equal areas of the orbit in equal times.

But now it must be evident that very few of our pupils either in high school or college are going to accomplish such results. Undoubtedly we do have in our classes some few students who will be the scientific discoverers of their generation but they are few and far between. The great majority will be consumers of science, not producers. They will need to apply the already known principles of science to the solution of the problems of every day life—problems which they will solve because they must do something of a practical sort about them or problems solved merely to satisfy their intellectual curiosity.

There follow some such problems:

The directions on this package of seeds I am planting say "Firm the earth about them after planting." What is that for?

Why does putting salt on weeds, kill them?

It is customary in many sections of the southern United States to plant sunflowers about the house. The current belief is that they prevent malaria. Is this just another nature myth or is there scientific foundation for the idea?

I notice that when I water the garden the soil appears much darker on the watered areas. Why?

You blow out the flame of a match yet you blow on a sluggish fire to make it blaze. How explain the apparent contradiction?

Why do patches of oil on the wet pavement appear so full of color?

I notice that after the container is open grape jelly keeps without fermentation much longer than grape juice. Why?

Why are fires built in orange groves when there is danger of a frost? Can such small smudges heat all outdoors?

Is it true that if it rains before seven it will clear before eleven?

It is said that "brains and beauty seldom go together." Is that true?

If you encountered such problems in a text book of science you would probably have less difficulty in solving them than

when stated here because you could look back in the chapter at the end of which they occurred to see what principle was discussed and you would know what principle to apply to the solution of the particular problem. In life, problems do not come labelled with the name of the proper principle to use in the solution.

How does one go about the solution of such a problem? He tries first to pick out the essential elements in the problem. Take, for instance, growing sunflowers as a preventative of malaria. The essential thing seems to be some relation of sunflowers and malaria. I hold these two things in the focus of my attention—sun flowers, malaria—to see if either will suggest a promising lead. Malaria recalls quinine. Can it be that growing sunflowers give off quinine or some substance like it which is inhaled by the person living near them. But quinine is bitter. I have no recollection of a bitter odor or taste from sunflowers. Moreover, I find from the encyclopedia that the plant from which quinine is derived is not at all related to the sunflower, so that the latter would not likely produce a substance similar to quinine. That seems to be the wrong trail.

I again focus attention on sunflowers, malaria. Malaria suggests disease, the germ nature of disease. Ah yes, I recall that the germ in this particular case is carried by mosquitoes. Now some relation of growing sunflowers and mosquitoes seems to be the crucial thing. Perhaps some insect lives on the sunflowers that gobble up the mosquitoes. That idea suggests the dragonfly which I remember is also called the mosquito hawk. But I do not recall that dragonflies are particularly abundant about sunflowers, rather are they found in swampy regions. So that hypothesis seems untenable.

So I hunt around in my head for some other suggestion. Sun flowers, mosquitoes—and now swamps occur to me as being the breeding places of mosquitoes. But swamps do not have any relation to sunflowers. Yet hold, a minute! Growing sunflowers do require a lot of water. This they absorb from the soil, send up to the leaves as a major part of the sap and give it off from the leaves. And I recall that plants give off immense quantities of water. I believe now I have it! The sunflowers dry up the soil so the mosquitoes can find no puddles in which to breed. That seems plausible and I accept it as the explanation and think it very likely that growing sunflowers may be an important adjunct in the prevention of malaria.

The essential elements in the problem are picked out and held in mind with the expectation that they will suggest some known principle, some relationship or sequence of events, that will serve to solve the problem. One may find that the things at first deemed essential are not the important elements. He may have to try several times before he hits the suggestive items. He tries out one principle to see if it will give an adequate explanation, then another if the first will not do, and so continues until the right one is found. The principle or principles are proven the right ones when they do enable one to solve the problem under consideration satisfactorily. Kepler solved his problem by the inductive type of thinking—the formulation of generalizations on the basis of facts. Here the deductive method is employed—bringing a specific problem under some known principle in order to solve it.

A teacher may guide the class in the attack on such a problem. Suggestions will come from this, that, and the other pupil. All must critically examine these. They must be rejected if they do not accord with the facts. Evidently fertility of hypothesis is an important element in the successful solution. Knowledge of a wide range of the principles of science is also needed. Pupils may have to read their texts on purpose to hunt for principles that will apply to the particular problem in mind. The solution must progress unhurriedly. The question could be raised as to how one could formulate experiments to test the validity of the conclusion. Suppose that living in a malarial region I do plant sunflowers about my house and none of my family have malaria. Would that be adequate proof that the conclusion reached above is correct? Suppose that in Marquette County, Michigan, I find no cases of malaria in houses that do not have sunflowers growing about them. Would that disprove the above conclusion? Suppose that in Rock County, Wisconsin I find fewer cases of pneumonia in homes with many evergreens about them than there are in homes not so protected. Would that have any bearing on the validity of the conclusion?

An attempt has been made above to make clear the distinction between inductive and deductive scientific thinking. The former is the method used commonly by the producer of science, the latter the method of the consumer of science. It is eminently desirable that skill in the deductive method be developed in public schools including colleges. A number of problems are suggested of the sort that can be used for this purpose.

The teacher must make pupils aware of the elements involved in scientific thinking and of the errors they are most likely to make. The pupils must practice thinking as they solve problems, preferably under careful supervision. To enumerate and discuss the elements and safeguards would transcend the limits of this article. They are presented in Chapter IV of the author's *Introduction to the Teaching of Science*.

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## WHY AND WHERE SHOULD THE SOLAR SYSTEM BE TAUGHT\*

By THEODOSIA HADLEY

*Western State Teachers College, Kalamazoo, Michigan*

In the first place it may be prudent to discuss why we approve of teaching any astronomy to children. Are we on sound ground when we say that children should be given the joy and the broadening view point that comes with knowing the stars?

Teachers have been agreed now many years on the necessity of preparing the child for leisure time. All of a sudden that leisure time is here. What are we doing to prepare the child to enjoy it?

Before the depression we were beginning to take the first steps toward our new responsibility—the education of the whole child—his body, his mind, his spirit. We are convinced that unless a child develops a healthy body he can not be a social asset unless he is a Steinmetz. If he does not have a trained mind he will be a social menace who will be swayed by any demagogue. Unless he has a balanced emotional nature and high ideals he will add nothing to society and will be a curse to himself. We need only cite the case of Loeb and Leopold.

The intelligent people who are interested in education believe in gymnasiums, physical examinations, music appreciation, art appreciation and nature appreciation. If these are frills of education they are the most important part of preparing the children to live a well rounded life in the twentieth century.

We appreciate the fact that our responsibilities as teachers have increased. The home, if it still exists, has pushed upon us not only the development of a sound body, a well balanced emo-

\* Read before the Elementary Science Section of the Central Association of Science and Mathematics Teachers, Dec. 1, 1933.

tional nature, a mind that is trained to think but now we are also faced with the added responsibility of developing the spirit of the child.

What subject in the curriculum can give a greater spiritual thrill, a longer stretch of the imagination, a more sound conception of the worth-while things that are of lasting value, a better balance of life in general, than an introduction to the wonders of astronomy? Interest in stars makes the best hobby for leisure time because they are with us where ever we are and it costs us nothing to enjoy them.

The development of astronomy shows us how we are progressing. The evolution of the idea of the universe has been enlarging with each step of civilization up the ladder of a finer and a better mankind. Do we have a tendency to smile at the naïve ideas of what the universe was supposed to be in the fifteenth century? A flat earth with the dome of heaven above and a very short distance below the sulfurous tortures of hell satisfied their scientific imagination.

What spiritual strides we have made since Galileo was jailed and put to the torture for saying that the sun was the center of the universe with the earth and the stars revolving about it.

The next step in the evolution of the idea of the universe is more thrilling to us today because it has happened so recently. Shapley's discovery of the island universes or galaxies, hundreds of millions of light years away, has stretched our imaginations to the breaking point. The thought that our galaxy, dragging our stars and our sun with his planets, on an orbit among the other galaxies, is overwhelming.

The last step in the evolution of the idea of the universe leaves most of us mystified. The theory of relativity and the exploding and expanding universe are as incomprehensible to us as Galileo's idea was to the Italians of his day.

When we are properly taught so that we can comprehend some of the wonders of nature it gives us a chance to appreciate the hollow value and the evenescent character of the material things that we have been so madly scrambling to obtain.

We teachers must grasp this opportunity to develop the spiritual side of the child so that the next generation may climb to the mountain top and look down at life and evaluate its various phases. It is our duty to develop a sufficiently well balanced person so that he can judge the material side of life for what it is worth and that the spiritual side of life will be sufficiently open

to him so that he will be able to grow in the enjoyment of lasting pleasures. We, as scientists and as teachers, can develop the spiritual side of life if we are permitted to give the child an introduction to astronomy. Letters have been coming from the east, middle west and south from teachers of nature who are now either teaching a grade or are out of school entirely.

Give the child the chance to thrill at the sight of the nebula in Orion. Give him the background so that his soul will expand and soar when he sees the fiery ball of Saturn whirling within his rings. Give his spirit the opportunity to get out and away from the narrow ruts, the sordidness of life, when he looks up at the heavens and appreciates what and where the stars are.

A weak link in the chain of our success is the teacher's colleges. Each teacher in the elementary grades and the high schools should have an astronomical background similar to that offered in the physical nature study work. By 1935 is it too much to ask that each teacher must have a college course in astronomy? Why can not we as scientists demand that one course in pedagogy be merged with the other courses in pedagogy and a course in astronomy be put in its place? Astronomy is much more in keeping with the trend of civilization in the twentieth century than our present over developed stress on theories.

A weakness of our meetings is that we do a lot of talking and then do nothing about it. Are we not sufficiently organized and are we not sufficient in numbers to make an impression on superintendents and school boards?

The above thoughts have shown why we may approve of teaching astronomy to children. They have shown that we are on sound ground when we demand it for the child.

As to where we should introduce the solar system and the more conspicuous constellations there is a diversity of opinion among expert teachers. Bertha Stevens of the Avery Coonley School, prefers to give it to the third grade children. It is interesting to hear her children tell about their experiences in looking at the stars.

Another nature teacher who stands at the top of our profession, Bertha Parker of the School of Education of the University of Chicago, gives it to fourth grade children. The children can not get to her room quickly enough and she has to push them out because they have so many questions to ask and so many arguments to finish.

Miss Guiney at the Highland Park School created so much enthusiasm among her sixth grade children that they interested the other children of the school. They all saved their money and bought a large telescope which is mounted in an observatory with a revolving dome. I asked Miss Guiney how often the children used the telescope. She answered in the quiet way that comes with four score and ten "Every night that it is clear the children come for me."

At Kalamazoo's Teacher's College we also think that the sixth grade child is at a level to get more out of a conception of the universe.

Gerald Craig of Teacher's College Columbia University puts a little astronomy in Book 2, a little more in Book 3, and still more in Book 6.

It makes little difference, however, where astronomy is taught or the methods of teaching it. Our chief concern is to use our influence in requiring all teachers to have sufficient astronomical background to teach the fundamental principles of astronomy and to be sufficiently thrilled with the wonders of the universe to make her enthusiasm contagious. Our next concern is to insist that each child in the elementary school shall be given enough astronomy so that when he gets out into the whirl of life he can look up at the stars and feel with Sara Teasdale:

#### STARS

Alone in the night,  
On a dark hill,  
With pines around me  
Spicey and still.  
And a heaven full of stars,  
Over my head,  
White and topaz  
And misty red.  
Myriads with beating  
Hearts of fire  
That eons  
Can not vex or tire.  
Up the dome of heaven  
Like a great hill  
I watch them marching  
Stately and still.  
And I know that I  
Am honored to be  
Witness  
Of so much majesty.

## NEW DEVICES USED IN THE TEACHING OF THE SEASONS

BY HERMAN SCHLEIFER

*James Monroe High School, New York*

A portion of the curriculum in general science is devoted to the teaching of the solar system. Naturally, our earth in relation to the sun is given a good deal of attention. We are interested chiefly in two motions of the earth, one a translatory motion about the sun, and the other a rotational motion about its own axis. The axis is tilted at an angle of  $23.5^{\circ}$ . Because the earth has its axis tilted and because of the types of motion previously indicated, we have seasons at various places on earth. At James Monroe High School we make use of the planetarium

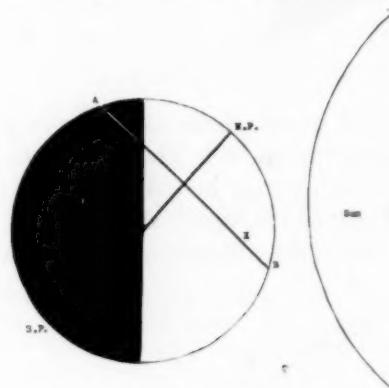


DIAGRAM A

in teaching this phase of the work. The planetarium consists among other things of a small globe representing the earth, which goes through the motions of translation and rotation. Another aid used is a blackboard sphere. At present we are working on additional devices, which we hope will clarify such concepts as the unequal length of day and night and seasons.

One way of treating the topic is to use diagram A, which indicates the position of the earth and its axis on June 21.

The great drawback in using this diagram is the fact that it is only the projection on a plane of a sphere and points and lines on the sphere. What is actually a small circle, like the path of point x about the axis, appears as a straight line AB in the projection.

A device we are attempting to develop, which will eliminate the indicated objection, is based on the principles of perspective. This is shown in diagram B.

A further device which is under way, that will supplement diagram B, is an actual model of diagram B plus some additions, as in diagram C, which may be made as follows:

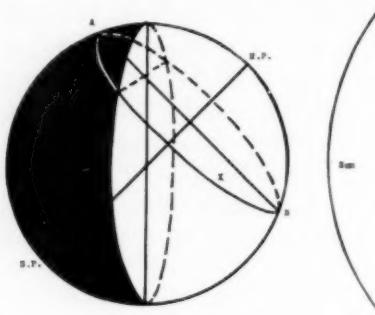


DIAGRAM B

A sphere made of wood whose diameter is about one foot, should be cut into hemispheres. The surface of one should be painted black. The other may be painted white or left as is depending on the color of the wood. After fitting the two hemispheres together, zones may be cut out, whose bases are perpen-

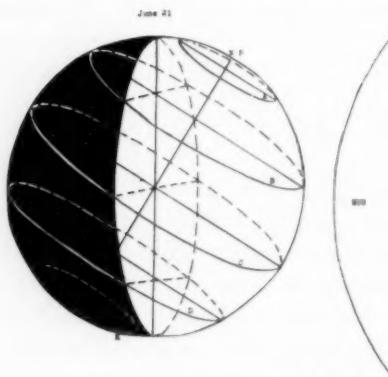


DIAGRAM C

- A. Grant Land
- B. New York
- C. Equator
- D. Brazil
- E. Mt. Melbourne

dicular to the earth's axis. A glance at diagram C will indicate appropriate places such as A, B, C, D, and E where these right sections might be taken. After this has been accomplished, pegs of some sort may be used to keep all the sections of the sphere together. Since diagram C refers to the position of the earth at

June 21, the one model may be inadequate. A similar model may be made for the earth at December 21.

The advantages of this type of model are as follows:

1. The sections into which the sphere has been cut are removable for demonstration purposes.
2. Areas of light and darkness appear on the surface of the sphere, which clearly indicate why there is a difference in the length of day and night of different localities.
3. The circumferences of the bases of the zones on the sphere illustrate the respective paths about the axis of various places on earth in a period of 24 hours.
4. Distorted notions of lengths of lines, and size of areas which diagrams may have left, are removed by the actual three dimensional models.

In conclusion I will briefly state what I think is the essence of the report, that the teaching of the seasons may be made more vivid by the use of solid models, in addition to charts and illustrations which show the positions of the earth both in projection and perspective.

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### RESEARCH AND THE SCIENTIFIC METHOD

By M. LUCKIESH, *General Electric Co., Cleveland*

Research is a profession which one learns just as he learns other professions. I presume a man interested in research must have aptitude, imagination, creative ability and the courage to be a pioneer. Eventually he learns how to perfect his method of attack so that his results are conclusive, that is, incontestable by not being vitiated by prejudice or some influence which has not been taken into account. I often liken him to trappers. He really sets a trap for results and the scientific method is merely a perfect trap. First of all, I believe it requires imagination to anticipate all the loopholes. Practice makes this anticipation more certain, that is, tends to perfect it. In scientific work no one sets out to obtain definite preconceived results. If he does this, his results are likely to be undependable or he is likely to be disappointed. Scientific method is merely one of obtaining results which are thoroughly dependable. Of course there is always a place for judgment before deciding to perform an experiment. We theorize to some extent but this must be done safely, that is, in a manner which does not introduce prejudice. Having a number of researches on our mind we naturally choose those which seem likely to produce valuable results. However, it is well to have the courage or abandon to attempt an experiment sometimes which is not promising or which at the beginning might seem to lead to disappointing results. The scientist learns that what may seem reasonable sometimes is not. The classic example is the case of the two unequal weights of equal size. Most persons unacquainted with the laws of gravity would believe that the heavier would fall the faster. This seems reasonable, but it is not true.

## EUCLID'S "ELEMENTS"\*

BY R. E. LANGER

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The invitation of your chairman to take part in this program confronted me with the task of choosing a subject with which I might hope in some measure to win your interest. It occurred to me that no few of my audience would be or would at some time have been teachers of geometry, and that, therefore, the story of the ancestry of those textbooks of which so many pages have so many times been turned might seem worthy of the time which you have placed at my disposal. This I propose to do, though I fear that for many of you I may be bringing nothing which is new, and as I am told it is often the custom of ladies to look first at the last page of a story, I will cast first a glance at that point of my narrative with which I shall in time reach a conclusion, namely, at the first printed English edition of the *Elements of Geometry* of Euclid. The title pages and introductory remarks of old books have for me a certain fascination. The urge of brevity now so universal is conspicuously absent from them, and in this respect the book I now have in mind is no exception. I will read to you a short abstract which has seemed to me interesting, and as I read you will picture to yourselves the orthography of the time three hundred and sixty-five years ago.

"The Elements of Geometry, of the most ancient Philosopher Euclide of Megara. Faithfully (now first) translated into the English tongue, by H. Billingsley, Citizen of London. Whereunto are annexed certain Scholies, Annotations and Inventions, of the best Mathematicians, both of time past, and in this our age.

"With a very fruitful preface specifying the chief Mathematical Sciences, what they are, and whereunto commodious; where also, are disclosed certain new Secrets Mathematical and Mechanical, until these our days, greatly missed."

"Printed by John Daye, dwelling over Aldersgate beneath S. Martins. These books are to be sold at his shop under the gate."

The translator says in part:

"There is, gentle Reader, nothing, the Word of God only set

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\* Read before the Mathematics Section of the Wisconsin State Teachers Association.

apart, which so much beautifieth and adorneth the soul and mind of man as doth the knowledge of good arts and sciences. . . .

"In histories are contained infinite examples of heroical virtues to be of us followed, and horrible vices to be of us eschewed. Many other arts also there are which beautify the mind of man; but of all other none do more garnish and beautify it than those arts which are called mathematical. Unto the knowledge of which no man can attain without the perfect knowledge and instruction of the principles, grounds and elements of geometry . . . ."

Interesting, yes even spectacular as has been the development of mathematics since the days of Billingsley I must pass it by, for even the briefest review of events prior to that epoch will I fear exhaust the time of the present occasion.

The origins of mathematical thought are veiled in the utter darkness of the remotest antiquity. Historical knowledge begins where men have left monuments or written records in stone, in clay, on wood, or on other materials, and in even the earliest such records there are evidences of mathematical consciousness, while the monuments in themselves speak of a developed sense of geometric form and regularity. The sources of knowledge concerning what men in ages past thought about are much more obscure than the sources of knowledge of what they did. The cogitations of the quiet and perhaps solitary thinker have not the immediate glamour of the exploits of a conqueror. Far less often and less conspicuously are they recorded. Yet the development of a type or mode of human thought is a matter profoundly interesting, and more especially so when, as in the case of mathematics, achievements won through the patience and toil of thousands of years and countless generations have shown themselves of such permanent value as to find a place in the educational plans even of today All too many are the things men have thought interesting or still think interesting and important which are answerable to the words of Mathew Arnold when he says: "There is not a creed which is not shaken, not an accredited dogma which is not shown to be questionable, not a received tradition which does not threaten to dissolve." In contrast to this mathematical attainments have come down through history passing from each race or nation as it dies or sinks into stagnation to the hands of another in which the spirit of life and achievement is young and fresh. In intellectual evolution the

civilized peoples of the world and of all ages are as a confederation bound together by a continuous and common aim and purpose. And in this great scheme of things it should be borne in mind that mathematics plays a rôle entirely like those of art and of literature. Its vitality and justification have never depended upon its applications but rather upon the fact that it is a primary medium through which men have found possible the expression of themselves, that it is a distinct form of high human achievement.

In so far as we know the earliest seats of mathematical thought were in the lands of the Euphrates and of the Nile. The records of the former have come to us scratched in curious form upon tablets of clay, while those of the latter stand engraved in hieroglyphics upon obelisks and pyramids, or stand in hieratic characters for us to decipher and read upon ancient scrolls of papyrus. The pyramids were at first built, not in the shape as we see them, but in the form of a succession of rectangular stories one above the other, and each of a smaller ground plan than that upon which it stood. The completion to their final shape required the cutting of stone into prismatic and tetrahedral forms to fill both the corners and the angular spaces between the stories. Neither the conception nor the execution of such a task could have been possible without the possession of a considerable body of geometric facts and a keen appreciation of geometric relations and proportions.

We have thus evidence that in very remote reaches of time a knowledge of geometry existed. Tradition has always set the place of its origin as in Egypt, and the cause of its birth the necessity for frequent surveys of land occasioned by the periodic inundations of the Nile. However materialistic this inception may have been, we have now positive knowledge that mathematics was at a very early time already the subject of purely intellectual activity. The advance of Egyptology in the last century has been great, and among the numerous finds in the ancient tombs and ruins of the land of the Pharaohs there have been many of mathematical importance. Preeminence among these belongs to a certain papyrus now in the British museum which bears the superscription "Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets." This scroll, written in the hieratic language is almost eighteen feet long, and contains, besides a rudimentary mathe-

matical table, a list of eighty-seven problems with their solutions according to the methods of its time.

Will you tarry with me briefly over this ancient record? Its geometrical problems deal with the volumes and proportions of parallelepipeds, prisms and cylinders, and with the ratios among the edges and altitudes of pyramids. In the plane they concern the areas of rectangles, of isosceles triangles and trapezoids and of circles. The value of  $\pi$  was taken to be the square of the number  $(1 + \frac{2}{3} + 1/9)$  and so was in error by less than  $\frac{2}{3}$  of one per cent. The analytic problems of the papyrus deal predominantly with the arithmetic of fractions, and, in many cases, are of a very considerable complexity. Their solutions often display astounding feats of skill, for it must be remembered that no number system of the perfection and pliability of ours, and no algebraic machinery of the power and subtlety of ours was at hand to these early mathematicians. Yet in the scroll there stands the solution of the problem to divide 100 loaves among 5 men with shares in arithmetic progression and such that  $1/7$  of the sum of the largest three shares shall equal the sum of the smallest two; and of the problem to find the quantity which when added to  $\frac{2}{3}$  of itself gives a sum of which in turn  $\frac{2}{3}$  is equal to ten; and there is a problem to which the answer is the number  $14 + \frac{1}{4} + 1/56 + 1/97 + 1/388 + 1/679 + 1/776$ . May I read to you the actual words of the papyrus in the case of just one of its problems. The 67th problem reads as follows: "Example of reckoning tribute." "The herdsman came to the stock-taking with 70 cattle. The accountant said to the herdsman 'Very few tribute cattle art thou bringing; pray where are all thy tribute cattle?' The herdsman replied to him, 'What I have brought is  $\frac{2}{3}$  of  $\frac{1}{3}$  of the cattle that thou hast committed to me. Count and thou wilt find that I have brought the full number.'" The solution which follows, begins with the customary phrase—"Do it thus."

Have I in any measure succeeded through this brief citation of fragments in persuading you that even in the days of this antique scroll mathematics had reached a stage transcending the merely practical? Yet the dating of the papyrus reads in part as follows: "This book was copied in the fourth month of the inundation season in the 33d year under the reign of the king A-user-Re<sup>6</sup>, in likeness to writings of old made in the time of the king of Upper and Lower Egypt, Amenemhêt III. It is the scribe Ahmes who copies this writing." It is thus shown that Ahmes

copied the papyrus which we have from another such which in his time was two hundred years old, whereas he himself lived seven centuries before the days of the splendor of King Solomon, or, to put it differently, as long before the siege of Troy as Columbus lived before us. The problems of the papyrus were solved at least as many years before the hey-day of the Roman Empire as we at the present are subsequent to that era.

It may perhaps be permitted me still, before leaving this manuscript of Ahmes, to mention a feature rather curious than significant. In summing a geometric progression with the ratio 7 which constitutes one of the problems, he permits himself to write the table

houses	7
cats	49
mice	343
grain	2401
measures	16807

The suggestion here is strong that the original scribe had in mind 7 houses, each of which had 7 cats, each of which killed 7 mice, each of which would eat 7 ears of grain which would each produce 7 measures at the harvest. He might equally well have written:

I met a man with seven wives,  
Every wife had seven sacks,  
Every sack had seven cats,  
etc., etc.,

and precisely as is suggested in that famous nursery rhyme he ignores utterly the precept which forbids adding together houses, cats, mice, etc.

The civilization of Egypt was a priestly one. The temple dominated the community, and the priest as custodian of the temple was held in awe and respect. He was the record keeper, the writer, the man of magic, and in the aloofness of his position he found the occasion for meditation. The intellectual life of Egypt thus grew to be the possession of the priesthood, jealously guarded and handed on only to those of the caste who had been trained and duly consecrated. And so it is not until a thousand years after Ahmes that we hear of the wisdom of Egypt being tapped by one of a foreign nation. Then, it was the time of captivity of the Jews in Babylon, Thales of Miletus, a Greek merchant and sage, is recorded to have journeyed to

Egypt and to have returned thence with the knowledge of Geometry. Thales was surnamed "the wise" and many geometrical facts are reputed to have been known to him, among them the theorems of the inscription of a right angle in the semi-circle, and the equality of the angles in an isosceles triangle. He is said to have deduced the heights of the pyramids from the lengths of their shadows, and to have measured the distances of ships at sea.

In the old age of Thales there came to him a youth by name Pythagoras, who was destined to be one of the most romantic and powerful figures in the history of ancient learning. He was urged by Thales to go to Egypt, and upon his arrival there he set himself the next to impossible task of penetrating to the innermost recesses of Egyptian knowledge. So well did he succeed that he is found years later to have himself become an Egyptian priest and to have risen to the highest rank within the order. The ancient nation threatened to have trapped the man who had so audaciously sought its possessions. At this juncture, however, an invasion of Egypt served to alter the course of events, for Pythagoras was carried off as a military prisoner and was detained as such for many years in Babylon. Babylon was a great and thriving city in which Hindoo and Persian, and even Chinese, rubbed elbows with Jew and Greek and Chaldean, and his enforced idleness in such surroundings was no futile challenge to the eager spirit of Pythagoras. When on his release he returned to his native country, Greece, at the age of fifty-six, he carried with him the finest parts of the entire knowledge of the world. The subsequent vicissitudes of his life were many. The years of his priesthood had not failed to shape his ideas, and in his teachings he retained the choicest morsels of his knowledge for those whom he admitted to a closely knit brotherhood, the Pythagoreans. His activities and interests were by no means confined to mathematics. He and his brotherhood fell ultimately into political disfavor. At the age of ninety-eight he was forced to flee his house which had been fired by an angry mob, and in the following, his ninety-ninth, year he died.

The mathematical achievements of Pythagoras were many. The beautiful theorem which bears his name is known to all of you. He is credited with the discovery of the irrational numbers, a fact of the greatest subsequent influence upon the trend of Greek thought, and to have founded the study of the theory of

numbers as distinct from the work-a-day arithmetic of practical life. Above all he is considered the true founder of the systematic speculative ideal of geometry. The geometry of Thales and of the Egyptians was a geometry of isolated concrete facts of experience. From the days of Pythagoras dates the ideal of the logical proof and of a structure in which the simpler leads on to the ever more complex. It is with reluctance that I pass on from him with so inadequate a discussion of his place in the history of our science.

Pythagoras lived but a few years beyond the time of the battles of Thermopylae and Platea, those events so crucial in the history of Greece, through which the defeat of Xerxes was encompassed and the dread of Persian domination was at last dispelled. The golden age of Athens commenced, when for a hundred and fifty years great writers, great builders, great sculptors, great philosophers and mathematicians followed one another in a continuous succession. Among the geometers—Hippocrates, Archytas, Plato, Eudoxus, Menaechmus, and many others. Time restricts me to the mere mention of their names. This is the period in which the geometry of our school-books was rounded out and perfected, and reasoning as the basis of proof was firmly and conclusively enthroned. The development of the theory of proportion and the discovery of the conic sections belongs to this period, as do also those classic problems of the duplication of the cube and the quadrature of the circle.

This brilliant period of the intellectual domination of Athens closed abruptly in the year 330 B.C. with the advent of Macedonia as a power in the world's history. The destruction of the city of Thebes by Alexander was a crushing blow to the Greek spirit, and though with the passing of his evanescent empire of conquest an Athenian rejuvenation might normally have followed, the circumstances of fate decided otherwise.

The Egyptians had welcomed Alexander, and he on his part was so fascinated with the ancient nation as to accept an Egyptian godship and to found upon the delta of the Nile the city to which he gave his name. Alexandria flourished from the start and when, on the death of its sponsor, Egypt fell to the lot of the ablest of Alexander's generals, Ptolemy, its future, intellectually as well as commercially, was assured. In the year 301 B.C. there was founded there the great university and library which were to dominate for many hundreds of years the spiritual

life of the world. Mathematically Athens never again approached importance, and Alexandria was to produce a succession of Greek geniuses which far outshone the greatest that had ever flourished on the native Greek soil.

The first mathematician at the new university was Euclid. We know nothing authentic of his private life. His influence upon the world is that of the disembodied teacher. As a mathematician he is of respectable stature though hardly great. As an organizer of existing knowledge, as an expositor, as a teacher in the best sense, he is without a peer. It is given to few men to be leaders during their lifetimes, and rare is he whose work and guidance outlives him by a century, but Euclid has taught for more than twenty-two centuries and still teaches in our time. He has taught not only at Alexandria but wherever on the face of the earth a civilization has shown itself. As late as the middle of the last century a mathematician of international distinction felt no hesitation in saying, "There never has been, and till we see it we never shall believe that there can be, a system of geometry worthy of the name, which has any material departures from the plan laid down by Euclid."

At Euclid's time three centuries had passed since Thales brought to the Greeks the geometry of Egypt. Through the work of a host of thinkers the Greek genius had made of it a great intellectual conquest, and had amassed a staggering volume of facts and theory. This volume was sifted by Euclid; the facts were arranged, the hypotheses enunciated with care, and the proofs unified in form. So effective was his work that in no single instance did even a single copy of an earlier work on geometry (and we know that there were many) survive the competition. Not until the most modern times has any subsequent work succeeded to any degree in displacing his. Of the "Elements" there exist in the world today over seventeen hundred distinct versions and editions, and no language of culture is without its representatives in the array.

The complete "Elements" as written by Euclid includes a far greater amount of material than is customarily read in it in the present time. It is written in 13 books, books 1 to 6 dealing with the geometry of plane figures and the theory of proportion, books 7 to 9 with arithmetic and the theory of numbers, in geometrical form as was customary with the Greeks, book 10 with quadratic irrational numbers, and books 11 to 13 with solid geometry. The division of a work into so large a number of

so-called "books" may perhaps strike you as curious. But a book in the days of Euclid was a manuscript upon a continuous scroll. The reader upon finishing the reading of a page rolled it up with the one hand while unrolling the succeeding page with the other. The wear and tear upon such a book was large, and reference to earlier pages of text was clumsy and laborious. These were cogent reasons for dividing a work which was of any substantial length.

Euclid wrote besides the Elements on other topics in mathematics, as well as on optics, astronomy, and music. Some of these books are known to us today; others are lost and we know merely that they once were in existence.

The greatest mathematical genius of the ancients was Archimedes who lived in the generation after Euclid's. His achievements have become legend, and in this fact I will console myself for passing by him so briefly here. As a geometer his achievements include the quadrature of the circle, namely, the development of a method by the inscription and circumscription of polygons about the circle for the computation of the value of the constant  $\pi$ . He found the area of the sphere, the inscription of the sphere in the cylinder, wrote books on the parabola, the conoids and the spheroids, and did many things more. Under his hand the first true development of infinitesimal analysis was achieved; and had there been able successors to him the calculus would not have had to wait for two thousand years for its effective development. The death of Archimedes came in the year 212 B.C. He was among those massacred at the fall of the city of Syracuse, after his ingenuity in the invention of means of defense had protracted for three years the Roman siege of the city.

I must hasten my narrative and but briefly mention Appollonius, the great geometer, whose work on conic sections will never fail to win the highest admiration; Heron, among the greatest of the Alexandrian school; and Eratosthenes who, even in his time, succeeded in measuring the size of the earth. Pythagoras had believed and taught that the earth was a sphere and that the sun and the moon were so likewise. Eratosthenes, we are told, observed at the city of Assuan on the Nile a deep well, the water in which reflected at noon the rays of the sun overhead. At Alexandria the sun remained a fiftieth of a revolution from the zenith, and so he concluded the circumference of the globe to be fifty times the distance between the two cities.

The great light of Greek learning which had thus been burning so brilliantly at Alexandria was quenched abruptly a hundred years before the advent of Christ under the tramp of the Roman legions. The Romans had neither understanding nor appreciation for the more abstract elements of Greek culture. So complete was their extermination of intellectual activity that a revival of learning at Alexandria did not occur for almost two centuries. Then, to be sure, we find again such men as Menelaos, Nichomachus, and Diophantes, and after them Pappus, Theon, and Proclus. But these last are no longer mathematicians in the earlier sense. The spark of genius was no longer abroad; the inspiration of the golden age was gone. From original thought men turned to a veneration of the masters of the past. Their works were compiled and edited and lengthy commentaries were written upon them. We owe these commentators a vast debt, for much of our knowledge of the preceding ages has come to us through them. Many of the great works of earlier days, though lost to us now, are described at length in the commentaries, which have thus preserved for us much material of great value.

In this group of earnest but less gifted men, Theon of Alexandria made a revision of Euclid which, though we consider it now in certain features a corruption of the original, attained an ascendancy in which it displaced almost all versions of earlier date. It is likely that a revision at that time was sorely needed. Though Theon taught at the ancient stand of Euclid, there had elapsed between the two men a span of time equal to that which separates us from the days of Marco Polo and the time of the great plague of the Black Death. Many are the manuscripts of the "Elements" which had been worn out and replaced and worn again in the intervening centuries. Changes, errors, misstatements, corruptions of all sorts must have become common in the versions of the time. Theon's daughter, also a mathematician, was Hypatia, whom you may know as the heroine of Kingsley's romance. She also taught at Alexandria, and for retaining her faith in the paganism of ancient Greece came to a tragic end. She was, you recall, set upon and torn limb from limb by a mob of Christian fanatics in that city with the illustrious past. It was just such a mob which destroyed finally the great library of manuscripts and so annihilated at last the queen of all universities.

These events were characteristic of the times. It was the fourth century of our era. The noon-day of creative thought

was long past, the chill of evening was at hand and the night of the dark ages stood before. The spirit of man no longer sought the legacy of the ancients, but strained for other things. The teachings of the church had made all other knowledge not alone superfluous but contemned. Of far greater importance the why and how that which the church taught must be true.

Earnest students have, to be sure, existed in the darkest of times. But the great works were no longer understood by them; things once commonly known became forgotten lore; mysticism took the place of science; the commentators, no longer the originals, were read. Boethius, the best mathematician ever to arise among the Romans, wrote a geometry, a sad affair when judged by any Greek standard, in which proofs were omitted as superfluous. This was the geometry of Europe which found its circulation in the monasteries, the only remaining seats of such learning as survived.

As Europe sank into this state of decadence and stagnation, there arose in the East a new prophet, Mohammed, and fired by his teachings, the Arabian nation, theretofore obscure and passive in the world's history, sprang into life. Egypt and Persia were rapid conquests and in the span of a century the sway of the new faith was extended from India to Gibraltar and over the Spanish peninsula. On the ruins of the ancient city of Babylon was erected a new city, Bagdad, and here the new nation took its seat. The Arabians, ethnologically close relations of the Jews, showed themselves an intellectually eager and a song loving race. The culture of the peoples they had conquered fascinated them; they made that culture their own. Learned men, be they Hindoos, Jews, Greeks or Romans, were drawn to their cities and under the patronage of the Caliphs became the teachers of the Arabs. In the reign of the Caliph Haroun-al-Rashid, the Caliph of Arabian Nights fame, the "Elements" of Euclid, the works of Archimedes and of Apollonius and of many other Greek masters were translated into Arabian, and so during the long sleep of Europe, during those many futile years of the crusades and the Norman conquest, this dusky race of the East held the custodianship of the intellectual treasures of the world.

The signs of a mathematical awakening of Europe appear in the thirteenth century, when the sporadic contacts of Christian monks with the Arabian universities in Spain, and the more regular commercial contacts of Christian and Moore, opened the

way to a renaissance. So Euclid's Elements are translated again from the Arabian into Latin. The renewed interest which follows prompts the search for and the discovery of left-over Greek originals. The art of printing steps in to hasten the awakening and in 1482, a decade before the voyage of Columbus, the first printed Euclid appears at Venice. The stride increases and in less than a hundred years we arrive at the first printed English Euclid, that of Mr. Billingsley of which I have already told you. The world was ready to pick up the thread where Archimedes had paid it down. Descartes, Newton and Leibnitz stood upon the threshold of the times.

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### CONCERNING RESPIRATION FERMENTS

By WERNER WENZELMANN

(from *Unterrichtsblätter für Mathematik und Naturwissenschaften*,  
No. 10, 1933)

The following paragraphs are translated and should be of great interest to biology teachers. The entire article of six pages is a valuable contribution.

"The physiological significance of reactions with carbon monoxide consists in the fact that the iron atoms of the ferments and of the hemoglobin are blocked by the carbon monoxide and that with sufficient concentration of CO they are unable to react with the oxygen. The work of the hemoglobin in the organism is to transport oxygen. In the lungs it takes the oxygen from the air, transports it through the blood vessels into the capillaries where the combination of oxygen and hemoglobin dissociates because of the diminished pressure of the oxygen. The oxygen diffuses into the cells, is taken up there by the respiration ferment and is transmitted by catalysis to the elements that oxidize. On the contrary the oxy-hemoglobin does not act as an oxidizing catalyst toward the respiration ferment. If on the way a part of the oxygen would be given off in this manner it would contradict the character of the hemoglobin as a transportation agent for oxygen. Now if the air we breathe contains sufficient amounts of carbon monoxide (it was previously noted that 1/10 per cent is sufficient) then the iron atom of the hemoglobin will be blocked by means of the CO, insufficient oxygen will be supplied to maintain normal respiration in the cells and the organism dies of suffocation.

"The concentrations of carbon monoxide which bring about the formation of CO-hemoglobin are not sufficient to restrict respiration in the cells, that is, to block the iron atom of the respiration ferment.

"An example will illustrate this. Naldane put mice into an atmosphere containing carbon monoxide. CO-hemoglobin was formed so that no more oxygen was transmitted and the animals suffocated. An increase of the oxygen pressure to two atmospheres caused no break-up of the CO-hemoglobin, but the oxygen physically dissolved in the blood increased ten times. This increase was sufficient to supply the necessary oxygen for respiration of the cells and the mice remained alive."—(Translated by W. F. ROECKER.)

## A DEMONSTRATION APPARATUS FOR THE COMPOSITION OF TWO SIMPLE HARMONIC CURVES

BY E. G. PLASTERER

*Huntington High School, Huntington, Indiana*

It is the purpose of this report to introduce a piece of apparatus which can demonstrate the composition of two simple harmonic motions two ways:—at right angles, and at parallels. The harmonic motions are produced by two 4-ft pendulums which are adjustable to desired ratios (Fig. 1). By means of thread

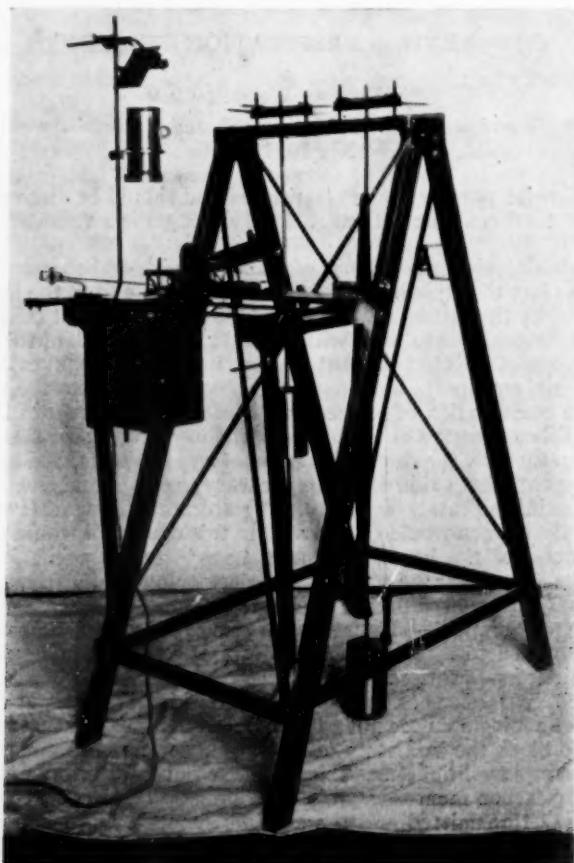


FIG. 1. Apparatus for a Demonstration in the Composition of Two Simple Harmonic Curves

connections, the two harmonic motions can be made to act upon a stylus at right angles and thus produce on smoked glass the well known Lissajous Curves.

To demonstrate the composition of the two motions at parallels, there are two styli, or markers, which move back and forth according to the motions of the pendulums. These two are called the component styli, and between these two is a third stylus

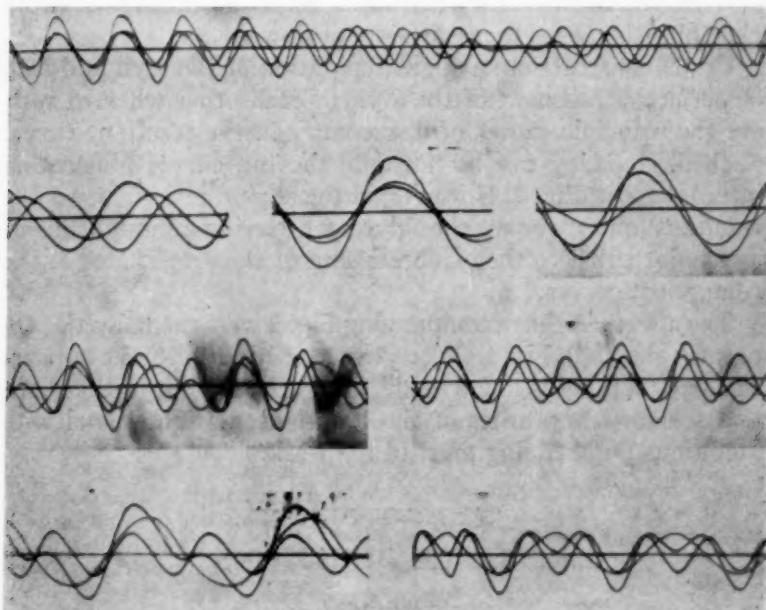


FIG. 2. The top curves illustrate interference and reenforcement. The second the composition of two curves of the same period, illustrating that for any difference of phase or amplitude the resultant has also the same period. The third shows the resultant of the ratios of 2:3 which correspond to C and G in music. The fourth illustrates the resultant to the Octave chord which has a ratio of 1:2.

which produces the resultant motion. When a smoked glass is moved along a track which passes under these markers, there is drawn on the glass the two component harmonic curves, the resultant curve, and also the normal line. The normal line is drawn by a fourth stylus which is stationary. By means of a lantern on the apparatus the respective motions for the styli as well as the curves produced can be projected upon a screen. When not in use the apparatus may be collapsed like a step ladder.

The apparatus yields itself to many distinct demonstrations

before either a few observers watching the styli directly, or before a large auditorium of people. The operator and an assistant may always have the apparatus under perfect control; for example, the pendulums can be slowly moved or held by hand and the results may be minutely and technically studied, or any combination of the four styli may be caused to trace the smoked glass, or the relative wave-length to the amplitude of the curves, may be varied merely by turning a crank, so that the smoked glass plate is caused to move faster or slower.

Crude construction, a slight separation of the styli and lack of perfect adjustment of the styli to each other when at rest, are the principle causes of discrepancy in the resultant curve. Such discrepancy can be noted in the top curves illustrating interference in Fig. 2. However, in the second row of curves the resultant appears as we should expect, having an amplitude at any point equal to the algebraic sum of the amplitudes of the component curves.

The curves in the accompanying Fig. 2 were made by the apparatus shown in Fig. 1. As one can readily surmise, the apparatus here pictured is home made, having been built in the shops and science laboratories of the Huntington High School with ambitious pupils doing most of the work.

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## SCIENCE QUESTIONS

April, 1934

Conducted by Franklin T. Jones, 10109 Wilbur Avenue,  
Cleveland, Ohio

*Readers are invited to co-operate by proposing questions for discussion and problems for solution.*

*Examination papers, tests, and interesting scientific happenings are very much desired. Please enclose material in an envelope and mail to Franklin T. Jones, 10109 Wilbur Avenue, Cleveland, Ohio.*

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## HOW MUCH IS BUNK?

The N.E.A. Department of Superintendence and the Progressive Education Association have just completed meetings in Cleveland.

Speakers took occasion to attack (as usual) the "domination of the secondary schools by the colleges," implying, to put it mildly, that the schools would be as well or better off without college entrance requirements.

So-called "college requirements" are better represented by questions asked on college entrance examinations than by any syllabus or outline. A College Board examination in Physics is given below.

**649. What questions here asked should pupils in an ordinary school physics course *not* have been able to answer satisfactorily?**

How much of the attacks on College Entrance requirements are just plain bunk?

(TWO HOURS)

*Answer ten questions as indicated below.*

*State the units in which each answer is expressed.*

*No credit will be given for problems on this paper unless the methods of reaching the results are clearly shown.*

*Number and letter each answer to correspond with the questions selected.*

### PART I

*(Answer all questions in this part.)*

1. a) Define energy, and name two energy units.  
b) If the speed of an automobile is doubled, is its kinetic energy doubled? Explain.  
c) Give an example of heat transformed into mechanical energy and an example of work transformed into heat.
2. A solid object is found by experiment to lose 200 grams when weighed just under water.  
a) State two conclusions which can be drawn from this result.  
b) What would be the result of the experiment if the object were immersed to a considerable depth in the water? Explain.
3. Five horses are pulling logs weighing 4,000 pounds along a level road. The coefficient of friction between the logs and the road is 0.25.  
a) With what total force do the five horses pull the logs?  
b) Assuming that each horse delivers one horse power, at what speed can the five horses pull the logs?
4. A kilogram of good soft coal evolves about 8 million calories of heat during combustion. How many grams of ice at a temperature of  $-10^{\circ}$  C. could be converted into steam at  $100^{\circ}$  C. by the combustion of half a kilogram of such coal? (Assume the specific heat of ice to be one-half that of water.)
5. Two identical lamps are connected in series on a 120-volt circuit, and together they use 20 watts. Find:  
a) the potential difference between the terminals of one lamp;  
b) the current through each lamp;  
c) the resistance of each.  
d) If they were replaced by a single lamp also using 20 watts, what would be the current and resistance for this lamp?
6. A toothed wheel has 65 teeth on its rim and revolves 240 times per minute. Find the frequency and wave-length of the note produced when a card is held against the teeth of the wheel.
7. The textbooks say that the image formed by an object placed in front of a plane mirror is virtual and as far behind the mirror surface as the object is in front.  
a) What is meant by a "virtual" image, and how does it differ from other images?  
b) With the aid of a labeled diagram describe an experiment to prove the statement about distances.

### PART II

*(Select three questions from this part. If more than three are answered, only the first three will receive credit.)*

8. A boy is to hold in a horizontal position a uniform pole 8 feet long. He can do it with one hand provided he place that hand at a certain point on the pole and exert a *vertical* force of 10 pounds.  
 a) What is the weight of the pole, and where on the pole is the "certain point"?  
 If, however, he places his right hand at the right end of the pole, and his left hand at a certain other point, he must exert a vertical force of 20 pounds with his left hand.  
 b) Where is the "certain other point," and what is the direction and magnitude of the force exerted by his right hand?
9. A lead ball is dropped from a height of 64 feet, and at the same instant a like ball is thrown upward from the ground with an initial velocity such that it rises just 64 feet. How far from the ground will the balls pass each other? After how many seconds will this occur?
10. a) State the present theory of heat.  
 b) In terms of this theory, explain the phenomena of thermal conduction, evaporation, and freezing.
11. What will it cost to use an electric heater to raise the temperature of 20 liters of water from  $20^{\circ}$  C. to  $50^{\circ}$  C.? The following additional information is given, although some of it is unnecessary:  
 a) The heater is on an alternating current line.  
 b) The difference of potential between the terminals of the heater is 120 volts.  
 c) Electrical energy costs 8 cents per kilowatt-hour.  
 d) Not more than 15 minutes is allowed for the heating.  
 e) One joule (watt-second) is approximately 0.24 calories.  
 f) Heat losses to containers and other surroundings are negligible.
12. a) What is meant by the statement that a certain specimen of glass has "an index of refraction of 1.50"?  
 b) In a table of indices of refraction which of these three, 0.75, 1.33, and 2.13, would you not expect to find? Why?  
 c) How far will light travel in air in the same time that it takes to travel 5 centimeters through a piece of glass of index of refraction 1.50?

### WHICH IS COLDER?

*In the March, 1934, number of SCHOOL SCIENCE AND MATHEMATICS Alden M. Shofner asked about the reading of the thermometer frozen in a block of ice (Question 645) when the air temperature is  $-10^{\circ}$  F.*

*Here is another question along the same line.*

650. Ohio has had an unusually severe winter with successive days on which the temperature has been away below zero.
- a—There is ice about 22 inches thick on Lake Erie. What is the temperature registered by a thermometer frozen in Lake Erie ice when the air temperature is  $-20^{\circ}$  F.?
- b—A pail of water was left standing beside the iron pump overnight when the temperature dropped to  $-15^{\circ}$  F. The ice froze solid and bulged the pail. What temperature would you expect the iron pump to show, the pail, the cake of ice in the pail? Why?
- c—A maple tree stood in the same yard. What temperature would you expect in the heart of the tree?

**651. Proposed by Alden M. Shofner, Shelbyville, Tenn.**

Some old soldiers tell me that they made their bed on the cold dry ground, on some quilts, and covered over with quilts, and while sleeping came a big snow, and soon the weather changed hurriedly and went to 10 degrees below zero, but that they slept warm till late next morning. Now the snow kept off the coldness, why not ice?

**THE SECRET OF LIFE****642. Suggested by Annual Report of the Carnegie Institution of Washington, Dr. C. B. Davenport, Head, Department of Genetics.****1. What is the "Secret of Life"?**

"The elusive secret of what life is was traced to the heart of the tiny gene."

**2. What is the "gene"?**

"The unit of heredity. The gene is the actual creator of life in addition to its better-known task of carrying inherited characteristics like color of eyes from one generation to another."

**3. Of what does the gene consist?**

"This life-creator probably consists of a compound molecule, a bundle of several smaller molecules which in turn consist of clusters of atoms. They create life by their ability to divide into other groups of molecules, all of which retain the qualities of the parent molecules."

"The living, dividing molecule, of which the gene is a special sort, is the great upbuilding agency in a universe that is running down."

**4. How large is the gene?**

Less than 8/1,000,000 of an inch in diameter.

**5. Can it be photographed? Why or why not?**

"Yes," when the proper apparatus has been devised. Not yet because of size.

**LIBERTY ANSWERS OWN QUESTIONS****638. Liberty Magazine, 1926 Broadway, N. Y. paid \$1.00 each for the following questions which were accepted for publication.**

1. Name the membrane which covers the lungs.

2. What is a "picul"?

3. What is a photometer?

4. What country bounds Denmark on the south?

5. What is an invertebrate?

6. What is a pluviometer?

*Answers from "Liberty" Magazine*

1. Pleura. 2. Picul—a varying weight used in Oriental countries. 3. Photometer—an instrument for measuring the intensity of light. 4. Germany. 5. Invertebrate—an animal with no back bone. 6. Pluviometer, a rain gauge.

**GQRA (GUILD QUESTION RAISERS AND ANSWERERS)**

**(Qualifications for Membership: At least one contribution per year.)**

**652. J. C. Packard, Brookline, Mass.** "Here's my entrance fee for 1934."

Two boys are pulling in opposite directions, with a force of 200 lb. each upon a rope stretched between them. What is the tension on the rope? If one of the boys increases his pull to 300 lb., while the other boy keeps his at 200 lb., what, then, will be the tension on the rope? Explain.

**ANOTHER MONKEY PROBLEM**

B. Felix John has sent the *Editor* another "monkey problem." If you want it published, write at once and say so to the Editor of Science Questions, 10109 Wilbur Avenue, Cleveland, Ohio.

**IS THERE JUST ONE SCIENTIFIC ATTITUDE**

By GEORGE J. SKEWES  
*University High School, Madison, Wisconsin*

The article in the March 1934 number of *SCHOOL SCIENCE AND MATHEMATICS* (Vol. XXXIV, p. 302) by Mr. Downing on "The Scientific Attitude and Skill in Thinking" demands comment for it tends to confuse the whole question of scientific attitudes, and also tends to confuse the question of scientific thinking with the scientific method of solving problems. Mr. Downing takes exception to an exposition previously made (*SCHOOL SCIENCE AND MATHEMATICS* 33: 964-68, Dec. 1933) on the scientific attitude, and he sets forth his own definition of scientific attitude. In order to clarify the thesis of the article to which Mr. Downing objects, the following facts are presented:

1. Various writers have attempted to define "scientific attitude."
2. These definitions differ in important aspects. As proof of this the reader is asked to compare Mr. Downing's definition of scientific attitude with any other definition.
3. No one definition is generally accepted by everyone.
4. Confusion and misunderstanding result from the use of the term "scientific attitude."

In order to explain these facts the hypothesis was advanced that "a scientific attitude differs with the situation; that there are really a number of component elements, using the generic name, and that some of these may be called forth at one time and some at another."

In order to test out this hypothesis the Wisconsin State Science Committee attempted to find out what the workers in the field of science teaching understood by and implied in their use of the term "scientific attitude." Unfortunately a chemical analysis cannot be made of a human brain in order to learn the general connotation of a term. If one wishes to know what people mean by a term he asks them what they mean and if possible phrases his questions in an objective manner. This explains why a questionnaire was used by the committee.

The returns from the questionnaire study supported the hypothesis advanced. People generally think of more than one element when the term "scientific attitude" is used. Therefore it is proposed that we clarify our writing and our thinking by substituting for the general question "Has he a scientific attitude?" such specific questions as the following:

1. Is he willing to change his opinions on the basis of new evidence?
2. Does he search for the whole truth regardless of personal, religious or social prejudice?

3. Does he have a concept of cause and effect relationships?
4. Is he in the habit of basing judgments on fact?
5. Does he distinguish between fact and theory?

When specific questions such as these are asked it is less likely that the question will be misunderstood. One cannot answer all these questions concerning a man by any "sole criterion" such as that suggested by Mr. Downing. Fortunately the answers to many of these questions can be determined by objective means.

The members of the Central Association of Science and Mathematics Teachers as well as the teachers represented by the Wisconsin State Science Committee are interested primarily in the benefits which science teaching can confer on boys and girls. It is not expected that high school science students will be "pure scientists" but there are certain attitudes which they should secure. If science teachers can agree on a number of the specific scientific attitudes which should be developed and can devise measures of the presence of these attitudes in boys and girls, then the efficiency of science courses in developing desirable attitudes can be determined.

Thus far nothing has been said about the method to be used in solving scientific problems. The best method to use is a question of fact which cannot be settled by asking for opinions. Mr. Downing's suggestion that a man should solve his scientific problems by reflective thinking does not accord with the usually accepted scientific method unless I misunderstand Mr. Downing's implication. It is true that the methods used by successful scientists of the past should be studied as examples of successful methods, but care must be taken not to generalize from a single case such as Roemer's discovery of the speed of light or Ehrlich's #606, Salvarsan, cure for syphilis. Perhaps the method used by Newton in determining the law of gravitation would not be applicable in helping a man decide which water softener to buy.

The question of how to teach the scientific method in high school science must be faced by science teachers. In facing this problem teachers must show the same specific scientific attitudes which they hope to develop in pupils in their classes. Can we be consistent?

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## PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON

*State Teachers College, Kirksville, Mo.*

*This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.*

*All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution as on the following pages.*

*The Editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.*

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### SOLUTIONS AND PROBLEMS

**Note.** Persons sending in solutions and submitting problems for solutions should observe the following instructions

1. Drawings in India ink should be on a separate page from the solution.
  2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.
  3. In general when several solutions are correct, the one submitted in the best form will be used.
- 

### LATE SOLUTIONS

**1306.** *E. B. Worthington, Lancaster, Pa.*

**1316.** *Proposed by Franklin T. Jones, Cleveland, Ohio.*

An automobile license plate carries five figures, all ones or threes. If one of the figures was one greater, the number would be divisible by 31, and the quotient would be all ones or zeros. In the quotient there are three figures alike. What is the number on the license plate?

*Solved by John E. Bellardo, St. Nazianz, Wis.*

The quotient from the condition of the problem must be 1110, 1000, 1011, or 1101. The last two are the only ones to be considered. Multiplying 1011, and 1101 by 31, we obtain 31343 and 34131. Reducing the number 4 by one we obtain 31333 and 33131 respectively. The license may be either of the two numbers.

*Also solved by Charles W. Trigg, Los Angeles, Calif.; W. E. Bunker, Leetsdale, Pa.; Herman O. Makey, Fort Wayne, Ind.; and H. Hansen Smith, Battle Creek, Iowa.*

**1317.** *Proposed by Charles W. Trigg, Cumnock College, California.*

There are just three proper fractions,  $26/65$ ,  $16/64$ ,  $19/95$ , with denominators less than a hundred which may be reduced to lowest terms by illegitimately cancelling a digit. Are there any other proper fractions with denominators less than a thousand which may be reduced to lowest terms by illegitimately cancelling one or more digits? If so, identify them and prove that there are no others.

*Solved by W. E. Bunker, Box 66, Leetsdale, Pennsylvania*

The original of this problem, that of finding the three fractions with denominators less than a hundred was easily solved by finding solutions in integers of one digit of indeterminate equations of first degree in three unknowns. It is possible to solve the present problem in a similar manner theoretically; but practically, the solution of the indeterminate equations involving as many as five unknowns is too laborious. I found the following solutions by writing the denominators in factored form and trying numerators which looked promising. There are probably other solutions besides the ones given.

(1) Solutions whose denominators and numerators are multiples of 11.  
 $22/121 = 2/11$ ;  $44/143 = 4/13$ ;  $55/154 = 5/14$ ;  $77/176 = 7/16$ ;  $88/187 = 8/17$ ;  
 $55/253 = 5/23$ ;  $253/352 = 23/32$ ;  $77/275 = 7/25$ ;  $176/275 = 16/25$ ;  $187/286 = 17/26$ ;  $44/341 = 4/31$ ;  $143/341 = 13/31$ ;  $242/341 = 22/31$ ;  $55/352 = 5/32$ ;  
 $253/352 = 23/32$ ;  $77/374 = 7/34$ ;  $275/374 = 25/34$ ;  $88/385 = 8/35$ ;  $187/385 = 17/35$ ;  $286/385 = 26/35$ ;  $55/451 = 5/41$ ;  $154/451 = 14/41$ ;  $253/451 = 23/41$ ;  $352/451 = 34/41$ ;  $77/473 = 7/43$ ;  $176/473 = 16/43$ ;  $275/473$

$=25/43; 374/473 = 34/43; 188/484 = 17/44; 385/484 = 35/44; 77/572$   
 $=7/52; 275/572 = 25/52; 473/572 = 43/52; 88/583 = 8/53; 187/583$   
 $=17/53; 286/583 = 26/53; 385/583 = 35/53; 484/583 = 44/53; 77/671$   
 $=7/61; 176/671 = 16/61; 275/671 = 25/61; 374/671 = 34/61; 473/671$   
 $=43/61; 572/671 = 52/61; 187/682 = 17/62; 385/682 = 35/62; 583/682$   
 $=53/62; 88/781 = 8/71; 187/781 = 17/71; 286/781 = 26/71; 385/781$   
 $=35/71; 484/781 = 44/71; 583/781 = 53/71; 682/781 = 62/71$

(2) Solutions read off from the example given in the statement of the problem.

$$166/664 = 1/4; 266/665 = 2/5; 199/995 = 15$$

(3) Solutions in which the first two or last two digits of numerator and denominator cancel.

$$\begin{aligned} 412/721 &= 124/217 = 4/7; 424/742 = 244/427 = 4/7 \\ 448/784 &= 484/847 = 4/7; 436/763 = 364/637 = 4/7 \\ 218/981 &= 182/819 = 2/9; 327/872 = 273/728 = 3/8 \\ 545/656 &= 455/546 = 5/6 \end{aligned}$$

The fractions of this type are remarkable in that their value is not changed by a certain change in the order of digits in numerator and denominator.

Thus:  $436/763 = 364/637 = 4/7$

(4) Two digits in numerator, both cancelling

$$21/126 = 1/6; 88/688 = 1/8$$

(5) Miscellaneous

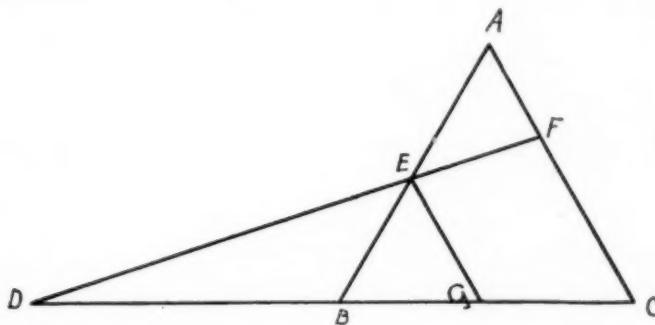
$$163/326 = 316/632 = 1/2$$

$$145/435 = 1/3; 139/973 = 1/7; 127/762 = 1/6; 187/749 = 1/4$$

It is possible that the solutions listed are not exhaustive for any of the types.

**1318.** Proposed by O. F. Barcus, Columbus, Ohio.

Extend one side, say  $CB$ , of equilateral triangle  $ABC$  to any point  $D$ . From  $D$  draw  $DF$  intersecting  $AB$  at  $E$  and  $AC$  at  $F$ . Show that triangle  $AEF$  is  $\frac{1}{3}$  triangle  $ABC$  when



- (1)  $DB = 0$  and  $AF = \frac{1}{3}AC$
- (2)  $DB = 2BC$  and  $AF = 2/3AC$
- (3)  $DB = 14BC$  and  $AF = 7/10AC$
- (4)  $DB = 84BC$  and  $AF = 12/17AC$ .

Solved by G. W. Grotts, Irving, Illinois

Let  $p$ ,  $q$ , and  $r$  be the ratios of  $BD$ ,  $AF$ , and  $AE$ , respectively, to a side of triangle  $ABC$ . Take the side as unity.

Draw  $EG$  parallel to  $AC$  forming equilateral triangle  $EBG$  of side  $1-r$ . From similar triangles:



$\frac{DG}{DC} = \frac{GE}{FC}$ , where  $DG = DB + BG = p+1-r$ ,  $DC = DB + BC = p+1$ ,

$$GE = 1-r, FC = AC - AF = 1-q$$

Then, substituting in terms of  $p, q, r$ , we obtain

$$\frac{p+1-r}{p+1} = \frac{1-r}{1-q}$$

Solving for  $r$ ,

$$r = \frac{q(p+1)}{p+q}$$

Furthermore

$$\frac{\Delta AEF}{\Delta ABC} = \frac{AE \cdot AF}{AC \cdot AB} = rq$$

When  $p=0, 2, 14, 84$  and  $q=\frac{1}{2}, \frac{2}{3}, \frac{7}{10}, \frac{12}{17}$ , from (1), (2), (3), and (4)

$$r=1, \frac{3}{4}, \frac{5}{7}, \frac{17}{24}$$

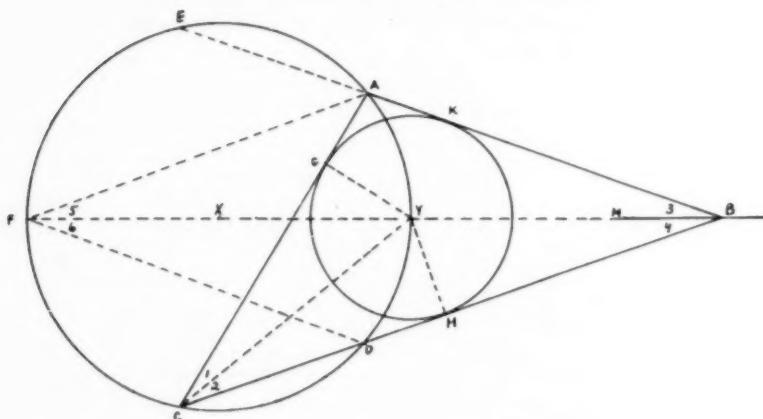
Thus, for each pair of values,  $rq = \frac{1}{2}$  and the proof is complete.

Also solved by Roy MacKay, Albuquerque, N. M.; Aaron Buckman, Brooklyn, N. Y.; Charles W. Trigg, Los Angeles, Calif.; W. E. Baker, Leetsdale, Pa.; and the proposer.

1319. Proposed by W. E. Batzler, Battle Creek, Mich.

Given two circles  $X$  and  $Y$  with center  $Y$  on the curve of  $X$ . A triangle  $ABC$  is a variable triangle having its three sides tangent to the circle  $Y$  and its vertices  $A$  and  $C$  on circle  $X$ . Find the locus of vertex  $B$ .

Solved by Aaron Buckman, Brooklyn, N. Y.



Given: circle  $X$  and circle  $Y$  with center  $Y$  on the circumference of circle  $X$ .  $AC$  is a chord in circle  $X$  tangent to circle  $Y$  at  $G$ .  $BA$  and  $BC$  are tangent to circle  $Y$  at  $K$  and  $H$  respectively.  $BC$  cuts circle  $X$  in  $D$  and  $C$ .

Required: the locus of point  $B$  as the position of  $AC$  is varied. Extend  $BA$  to cut circle  $X$  at  $A$  and  $E$ . Draw  $CY$ . Draw  $BY$  cutting circle  $X$  at  $F$ . Draw  $FA$ ,  $FD$ ,  $YG$ ,  $YH$ .

- (1)  $CG$  and  $CH$  are tangent to circle  $Y$ .  $\therefore \angle 1 = \angle 2$  and  $\widehat{AY} = \widehat{DY}$
- (2) Similarly  $\angle 3 = \angle 4$ , and  $\angle 5 = \angle 6 \therefore$  triangles  $FAB$  and  $FDB$  are congruent and  $\angle FAB = \angle FDB$ .

- (3)  $FEA + AY = FCD + YD$
- (4)  $FEAY = FCDY$
- (5) But  $FEAY + FCDY = 360^\circ$
- (6)  $\therefore FEAY = FCDY = \text{semi-circle}$
- (7)  $\therefore FY$  is a diameter of circle  $X$
- (8)  $\therefore X$ , the center of circle  $X$ , is on  $FY$
- (9)  $\therefore B$  is on  $XY$ , the line of centers of circles  $X$  and  $Y$   
Let  $M$  be the intersection of the common external tangents to circles  $X$  and  $Y$ .
- (10) Then  $B$  can not be between  $Y$  and  $M$ , for a tangent to circle  $Y$  from such a point does not cut the circle  $X$ .
- (11) The locus of  $B$  is the line of centers,  $XY$ , from point  $M$  to infinity.

Also solved by Roy MacKay, Albuquerque, N. M.

**1320. Proposed by Cecil B. Read, Wichita, Kansas.**

Each of the stories of a three-story house are of equal height. A ladder with its foot  $x$  feet from the foot of the building (on level ground) just reaches the top. A second ladder, raised from the same point, reaches only to the top of the second story. If the second ladder is  $y$  feet shorter than the first, determine the height of the building.

Solved by Hobson M. Zerbe, Wilkesbarre, Pa.

Let  $3s$  = the height of the building.

$l$  = the length of the longer ladder.

$l-y$  = the length of the shorter ladder.

Then:

$$\begin{aligned} (1) \quad & x^2 + 9s^2 = l^2 \\ (2) \quad & x^2 + 4s^2 = l^2 - 2ly + y^2 \\ (3) \quad & 5s^2 = 2ly - y^2 \end{aligned}$$

Since  $l = \sqrt{x^2 + 9s^2}$

Then  $5s^2 = 2y\sqrt{x^2 + 9s^2} - y^2$

$$(4) \quad 5s^2 + y^2 = 2y\sqrt{x^2 + 9s^2}$$

Squaring and combining like terms,

$$25s^4 + 10s^2y^2 + y^4 = 4x^2y^2 + 36s^2y^2$$

Solving for  $s^2$ :

$$(5) \quad s^2 = \frac{13y^2 + 2y\sqrt{25x^2 + 36y^2}}{25}$$

$$(6) \quad s = \frac{\sqrt{13y^2 + 2y\sqrt{25x^2 + 36y^2}}}{5}$$

$$(7) \quad 3s = 3/5\sqrt{13y^2 + 2y\sqrt{25x^2 + 36y^2}}$$

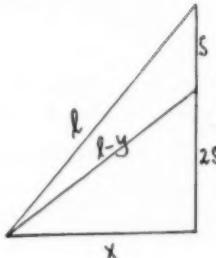
Also solved by Charles W. Trigg, Los Angeles, Calif.; W. E. Boker, Leetsdale, Pa.; Margaret Joseph, Milwaukee, Wis.

**1321. Proposed by H. D. Grossman, New York City.**

Find the fallacy:

$$\begin{aligned} \log 2 &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \\ \text{let } S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots \\ \log 2 + S &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \\ &\quad + 2(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots \\ &= S. \end{aligned}$$

Hence  $\log 2 = 0$ , which is impossible.



*Solved by Roy MacKay, Albuquerque, N. M.*

In view of the divergence of the harmonic series, no value of  $S$  exists which will make

$$S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

Hence the above demonstration merely illustrates a property of infinity; namely, an infinite quantity plus or minus a constant is still infinite.

*Also solved by John E. Bellards, St. Nazianz, Wis.; W. E. Boker, Leetsdale, Pa.; Aaron Buckman, Brooklyn; and Cecil B. Read, Wichita, Kan.*

### HIGH SCHOOL HONOR ROLL

The editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

For this issue the Honor Roll appears below:

1316. H. Hansen Smith, Battle Creek, Iowa.

### PROBLEMS FOR SOLUTION

1334. *Proposed by Cecil B. Read, The University of Wichita, Wichita, Kan.*

Given the simultaneous equations

$$\begin{aligned} x^2 + y &= 7 \\ y^2 + x &= 11 \end{aligned} \quad \left. \begin{array}{l} x = 2 \\ y = 3 \end{array} \right\} \text{one soln.}$$

Obtain a solution giving all values for  $x$  and  $y$ , without the necessity of solving any equation of degree higher than two.

1335. *Proposed by W. E. Boker, Leetsdale, Pa.*

In a shuffled pack of 52 cards, what is the probability that a four and seven (say) will be next to each other provided it is known that there is no four or seven among the first four cards?

1336. *Proposed by Charles W. Trigg, Cumnock College, Los Angeles, Calif.*

A horse is tethered by a 50 ft. rope to a ring fixed 30 ft. from the ground in the wall of a cylindrical silo, 20 ft. in diameter. Over what area can it graze?

1337. *Proposed by the Editor (suitable for high school pupils).*

The mid points of the parallel sides, the intersection of the non-parallel sides and the intersection of the diagonals of a trapezoid are collinear.

1338. *Proposed by Roy MacKay, Albuquerque, N. M.*

If  $P$ , a point on the Euler Line of the triangle  $A_1A_2A_3$  is one- $k$ th of the distance from the circumcenter to the orthocenter, then

$$PA_i^2 = [R^2(k-3)^2 + (a_i^2 + a_k^2)(k-1) - a_i^2]/k^2$$

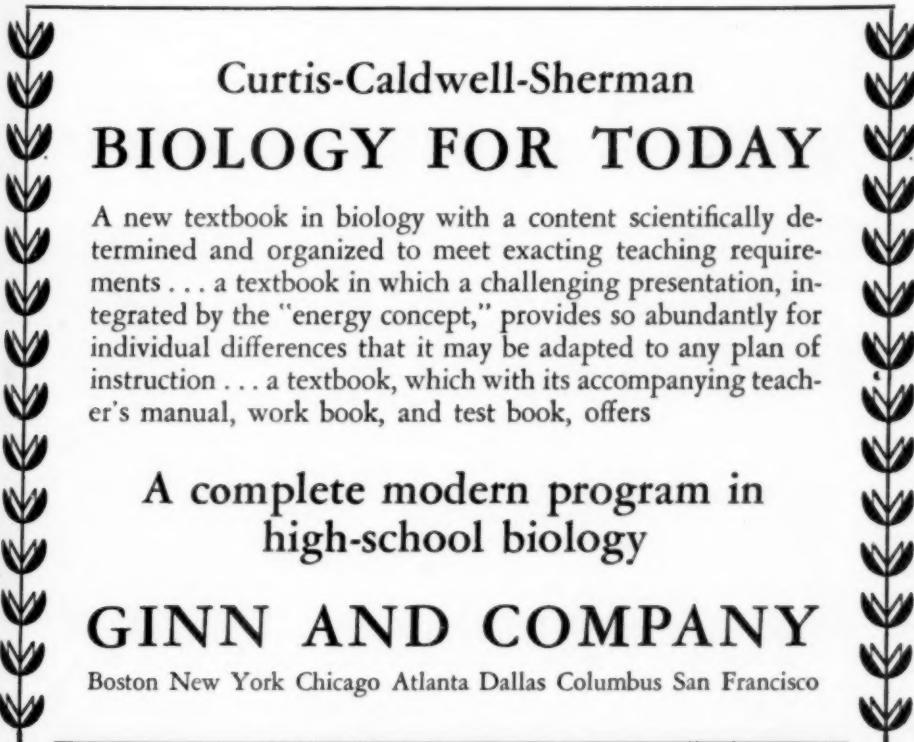
where  $a_i$  is the side opposite  $A_i$  and  $R$  is the circumradius of the triangle  $A_1A_2A_3$ .

1339. *Proposed by the Editor.*

Three numbers are in geometric progression. If three is added to the first, and two to each of the second and third, the resulting numbers are in arithmetic progression. Find the numbers.

**THE DAVY SAFETY LAMP***By LOUIS T. MASSON**Riverside High School, Buffalo, New York*

A suitable device for showing the principle of the Davy safety lamp can be made from two pieces of wire gauze such as is used in the laboratory with bunsen burners. Shape such a piece of wire gauze on a broom handle so that it will have the form of a cylinder and stitch the ends where they meet with fine copper or iron wire to keep the gauze in this shape. With a tin-shears, cut a circular piece of gauze out of the second piece, about  $1\frac{1}{2}$  inches in diameter. Stitch this as a top to the cylinder, and the safety lamp is complete. This device will enable one to illustrate the principle of the Davy lamp commonly used for this purpose. This is done as follows: Light a short piece of candle and place the gauze lamp over it. It will continue to burn unchanged. Direct a stream of *unlighted* illuminating gas into the side of the lamp, so that the gas strikes the flame of the candle. The gas will ignite and burn on the inside but will not strike back and light on the outside. The conductivity of the gauze is sufficient to remove the heat produced within the lamp and thus keeps the temperature of the gas below its kindling point. When the gauze gets red-hot, the flame passes through the gauze and ignites the gas on the outside. For schools that do not have illuminating gas, hydrogen could be generated in a small bottle and used in a similar way.



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### CHICAGO MATHEMATICS CLUB CELEBRATES TWENTIETH ANNIVERSARY

On January 19, 1934, the Men's Mathematics Club of Chicago and Metropolitan Area celebrated the Twentieth Anniversary of the founding of the Club. The fifty-five men present heard the recounting of past experiences, and of early efforts at charting the course in mathematics, and stories from and about the men that have become National Figures in the Field of Mathematics.

After a few remarks appropriate to the occasion, the President of the Club, Mr. Francis W. Runge of Evanston Illinois Township High School, turned the gavel over to Mr. C. M. Austin of Oak Park High School who remained Chairman for the evening. Mr. Austin was the first President of the Men's Mathematics Club of Chicago and Metropolitan Area during the period 1913-16. He was also the first President of the National Council of Teachers of Mathematics.

The Chairman's preliminary statements touched upon some early experiences of the Club and the nature of the meetings. He then directed the Secretary to read the letters that he had received from members who were unable to attend. Professor E. R. Breslich of the University of Chicago was unable to attend due to an important conference out of town that necessitated his presence. Mr. H. E. Cobb, formerly of Lewis Institute, but now retired near the "Rockbound Coast of Maine," sent his greetings. Mr. Alfred Davis of Soldan High School St. Louis, Missouri, regretted his inability to be present and wrote of a victory for required mathematics in the St. Louis High Schools.

We were greeted, via letter, from the State Normal School at California, Pennsylvania, by J. A. Foberg, who expressed a desire to be present at our party. Mr. G. A. Harper, who owns and administers the Southern Arizona School for Boys, wrote us from Tucson. He recalled past pleasantries and suggested that members of our Club who were looking for REAL work should open a private school in their old age. Mr. J. O. Hassler, Professor of Mathematics at the University of Oklahoma, desired to be present but business and distance prevented. Professor W. D. Reeve of Teachers College, sent warm greetings, recounted some pleasant experiences and wished our party success. From the University of Michigan we received a long and pleasant letter in which Professor Raleigh Schorling told some of his experiences while a member of our Club.

The reading of the letters was followed by speeches, poetry, stories, and jokes. These were related by past Presidents of the Club and by the "Old Timers" present. The verbal contributions of the many speakers were excellent but the stories told by Mr. E. W. Owen of Oak Park High School, and Mr. Marx Holt, Principal of a Chicago High School, "Of Peaches of Georgia and points South," stand out as entertainingly potent. It is interesting to note that only two of the past Presidents were not in attendance. These were Professor John R. Clark of Columbia University, and Mr. Olice Winter, Principal of Lake View High School, Chicago. Although they were not present, nevertheless, members who knew them well and knew of their work presented them to us in spirit.

The names of the past Presidents and the years during which they held the office follow: C. M. Austin, 1913-16; J. R. Clark, 1916-18; W. W. Gorsline, 1918-20; M. J. Newell, 1920-21; H. C. Wright, 1921-22; E. W. Owen, 1922-23; Olice Winter, 1923-24; E. W. Schreiber, 1924-25; Marx Holt, 1925-26; O. M. Miller, 1926-27; E. S. Leach, 1927-28; J. T. Johnson, 1928-29; W. H. Clark, 1929-30; Charles Leckrone, 1930-31; E. C. Hinkle, 1931-32; W. S. Pope, 1932-33. The present incumbent is Mr. Francis W. Runge.

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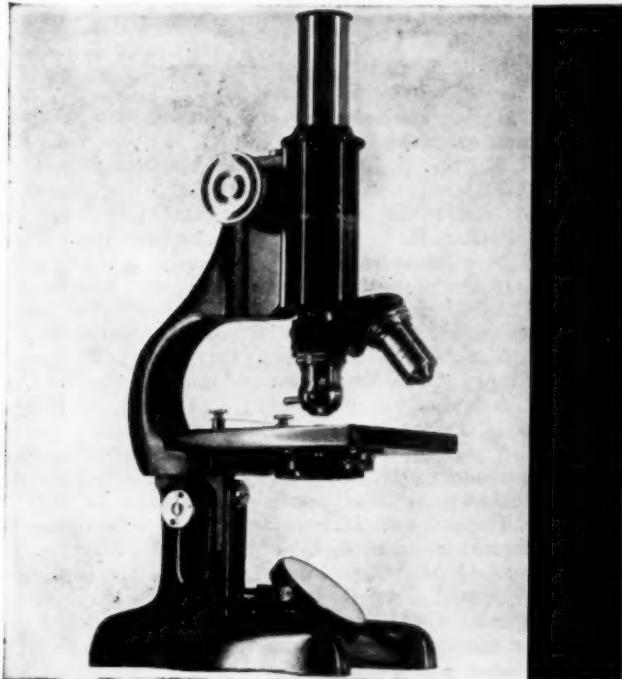
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During the first year (1913-14) the following men organized and became affiliated with the Club: C. M. Austin, E. R. Breslich, J. R. Clark, H. E. Cobb, S. J. Conner, W. W. Gorsline, G. A. Harper, Lawrence Irwin,\* C. E. Jenkins, F. A. Kahler, O. M. Miller, James Millis,\* G. W. Myers,\* M. J. Newell, W. D. Reeve, Harold O. Rugg, F. W. Runge, Raleigh Schorling, H. E. Slaught, W. A. Snyder, Olice Winter, and H. C. Wright. (\* Now deceased.)

In the years that followed other men became affiliated with the Club. The following list presents those that joined previous to the 1918-1919 season: A. M. Allison, A. W. Cavanaugh, M. W. Coultrap, Alfred Davis, J. A. Foberg, E. C. Hinkle, Marx Holt, J. T. Johnson, J. O. Hassler, J. M. Kinney, C. E. Kitch, Butler Laughlin, Charles Leckrone, D. W. Merrill, J. A. Nyberg, E. W. Owen, J. C. Piety, E. W. Schreiber, G. C. Staley, and G. G. Taylor.

A perusal of the foregoing list of names coupled with a knowledge of prominent mathematicians should indicate clearly to the reader the early field of professional operation of many.

The story of the evening would not be complete unless a list of those present is recorded. They were: W. H. Clark, C. E. Jenkins, Chas. Leckrone, O. M. Miller, Ross Herr, R. S. Woodruff, Claude E. Kitch, Everett W. Owen, Fred R. Rul, O. F. Rusch, O. E. Overn, W. S. Pope, M. J. Newell, C. M. Austin, Edwin W. Schreiber, F. W. Runge, S. F. Bibb, H. Sistler, Hans Gutekunst, C. E. Kellam, C. A. Jickling, F. P. Clyme, George V. Deal, E. A. Anderson, W. R. Matthews, Glenn Hewitt, A. W. Cavanaugh, F. R. Marks, F. M. Foster, H. W. Chandler, E. C. Hinkle, H. L. Sauer, Edgar S. Leach, H. C. Wright, V. B. Teach, W. C. Krathwohl, M. W. Coultrap, H. E. Slaught, Marx Holt, A. C. Lauder, H. S. Treese, Gordon M. Jones, H. W. Haggard, J. R. McDonald, Joseph A. Nyberg, Butler Laughlin, A. M. Allison, J. T. Johnson, D. Talmage Petty, A. S. Hathaway, J. D. Zmrbal, G. B. Reeve, W. W. Gorsline, E. M. Briggs, Joseph J. Urbancek. Total—55.—(Respectfully submitted, JOSEPH J. URBANCEK, Secretary-Treasurer, 1112 Grant Street, Evanston, Illinois.)

#### BOOKS RECEIVED

*Signals and Speech in Electrical Communication*, by John Mills, Author of *Within the Atom* and *Letters of a Radio Engineer to His Son*. Cloth. 281 pages. 12.5×19 cm. 1934. Harcourt, Brace and Company, Inc., 383 Madison Avenue, New York, N. Y. Price \$2.00.

*Mystery Experiments and Problems for Science Classes and Science Clubs* by J. O. Frank, Professor of Science Education and Head of the Chemistry Department, Wisconsin State Teachers College, Oshkosh, Wis., and assisted by Guy J. Barlow, Principal of the McKinley Junior High School, Appleton, Wis. Cloth. Pages ix+187. 12×17.5 cm. 1934. J. O. Frank and Sons, Oshkosh, Wisconsin, Price \$2.25.

*Electrons at Work, a Simple and General Treatise on Electronic Devices, their Circuits, and Industrial Uses*, by Charles R. Underhill, Consulting Electrical Engineer. First Edition. Pages xii+354. Cloth. 14×22.5 cm. 1933. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, N. Y. Price \$3.00.

*An Introduction to the Teaching of Science*, by Elliot Rowland Downing, The Department of Education, The University of Chicago. Cloth. Pages

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vii + 258. 13.5 × 19.5 cm. 1934. The University of Chicago Press, Chicago, Illinois. Price \$2.00.

*The Home Project in Homemaking Education*. Bulletin No. 170. Home Economic Series No. 16. Paper. Pages xii + 179. 14.5 × 23.5 cm. 1933. Superintendent of Documents, Washington, D. C. Price 15 cents.

*The History of Secondary Education in York and Oxford Counties in Maine*, by John Coffey Hylan, M.S. The Maine Bulletin, Vol. XXVI, No. 5, December 1933. Paper. 78 pages. 14.5 × 23 cm. University Press, Orono, Maine.

*The Habit of Scientific Thinking*, by Victor H. Noll, Research Associate, Institute of School Experimentation, Teachers College. A Reprint from Teachers College Record, Volume XXXV, Number 1, October 1933.

*Health Through the Ages*, by C. E. A. Winslow and Grace T. Hallock. Paper. 64 pages. 13.5 × 19.5 cm. 1933. School Health Bureau, Welfare Division, Metropolitan Life Insurance Company, New York, N. Y.

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#### BOOK REVIEWS

*A First Book in Chemistry*, by Robert H. Bradbury, Ph.D. Third Edition. pp. x by 633. 3 × 14.5 × 21 cm. Cloth. Illustrated. 1934. Appleton-Century Co. N. Y.

This third edition of Bradbury's *A First Book in Chemistry* has really been revised. The author is conservative, but while he would not be "the first by whom the new is tried" he is evidently not inclined to be "the last to lay the old aside." Much old material has been removed and new subject matter added. The generally accepted portions of sub atomic theory are given early enough so that much use can be made of them. The ionization chapter has been rewritten in the light of the modern idea of complete dissociation of strong electrolytes. The industrial side of chemistry has been treated more extensively and brought down to date. Teachers of chemistry in secondary schools will want to inspect this new edition of an excellent chemistry text.

F. B. WADE

*Mechanics and Applied Heat*, by S. H. Moorfield and H. H. Winstanley of the Mechanical Engineering Department, Wigan and District Mining and Technical College, England. Cloth. 328 + viii pages. 12 × 18.5 cm. 1933. Edward Arnold and Co., London. Published in the United States by Longmans, Green and Co., 55 Fifth Avenue, New York City. Price \$1.65.

*Mechanics and Applied Heat* is a textbook for Engineers of the Junior College level. The material is built around the requirements for the Ordinary National Certificate, analogous to our civil service requirements for government engineers. The principles essential to applied problems are given in a very logical and direct form. The theoretical material is developed without the aid of the calculus. Very little descriptive material is included. In contrast to the American texts no apparent effort has been made to make the text very readable and attractive.

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#### C. RADIUS

*A General Science Workbook*, by Chas. H. Lake, Louis E. Welton, James C. Adell. Silver, Burdett and Company. Revised 1932.

*A General Science Workbook*, embracing Health, Citizenship, Vocation, and Leisure as objectives, appears to have come closest realization in this production. The selection of science material, of vital interest to the pupil and of real intrinsic value, with apparently no sacrifice to accuracy or exclusion of fundamental scientific principle, is a tribute to the authors and the ideal for which general science teachers have been waiting.

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An important current topic which might have been more fully developed is the subject of fuels such as coal, and oil, and how to purchase them scientifically. Many interesting experiments could be formulated in this connection. I would also point out the rather interesting method of building a science vocabulary. The illustrations and diagrams labelled and to be labelled are especially well done and well selected.

A feature of the book which particularly appeals to the writer is that with the well selected specific references preceding each unit, it would be quite possible to use the set of reference books provided by a school library in conjunction with this book with no adopted textbook. While this procedure may not be highly desirable, it would be one possible economy, inasmuch as this book properly completed is really a valuable permanent possession for the pupil who does the work.

A significant feature is the scientific method whereby the pupil arrives at his conclusions and may make inferences. No progressive General Science teacher can afford to overlook this workbook.

#### F. R. BEMISDERFER

*College Physical Science*, by Paul McCorkle, Professor of Physics and Physical Science and J. Arthur Lewis, Professor of Chemistry and Physical Science, State Teachers College, West Chester, Penn. Cloth. Pages ix +327. 14×21 cm. 1934. P. Blakiston's Son and Co., 1012 Walnut St., Philadelphia. Price \$2.00.

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